

Chapter 9 Solutions

9.1 $m = 3.00 \text{ kg}$, $\mathbf{v} = (3.00\mathbf{i} - 4.00\mathbf{j}) \text{ m/s}$

(a) $\mathbf{p} = m\mathbf{v} = (9.00\mathbf{i} - 12.0\mathbf{j}) \text{ kg} \cdot \text{m/s}$

Thus, $p_x = 9.00 \text{ kg} \cdot \text{m/s}$ and $p_y = -12.0 \text{ kg} \cdot \text{m/s}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (-12.0)^2} = 15.0 \text{ kg} \cdot \text{m/s}$

$\theta = \tan^{-1}(p_y/p_x) = \tan^{-1}(-1.33) = 307^\circ$

***9.2** (a) At maximum height $\mathbf{v} = 0$, so $\mathbf{p} = \mathbf{0}$

(b) Its original kinetic energy is its constant total energy,

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} (0.100 \text{ kg})(15.0 \text{ m/s})^2 = 11.2 \text{ J}$$

At the top all of this energy is gravitational. Halfway up, one-half of it is gravitational and the other half is kinetic:

$$K = 5.62 \text{ J} = \frac{1}{2} (0.100 \text{ kg}) v^2$$

$$v = \sqrt{\frac{2 \times 5.62 \text{ J}}{0.100 \text{ kg}}} = 10.6 \text{ m/s}$$

Then $\mathbf{p} = m\mathbf{v} = (0.100 \text{ kg})(10.6 \text{ m/s})\mathbf{j}$

$$\mathbf{p} = 1.06 \text{ kg} \cdot \text{m/s} \mathbf{j}$$

***9.3** The initial momentum = 0. Therefore, the final momentum, p_f , must also be zero.

We have, (taking eastward as the positive direction),

$$p_f = (40.0 \text{ kg})(v_c) + (0.500 \text{ kg})(5.00 \text{ m/s}) = 0$$

$$v_c = -6.25 \times 10^{-2} \text{ m/s}$$

(The child recoils westward.)

***9.4** $p_{\text{baseball}} = p_{\text{bullet}}$

$$(0.145 \text{ kg})v = (3.00 \times 10^{-3} \text{ kg})(1500 \text{ m/s}) = 4.50 \text{ kg} \cdot \text{m/s}$$

$$v = \frac{4.50 \text{ kg} \cdot \text{m/s}}{0.145 \text{ kg}} = \boxed{31.0 \text{ m/s}}$$

***9.5** I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm.

I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a(x - x_i)$$

$$0 - v_i^2 = 2(-9.80 \text{ m/s}^2)(0.250 \text{ m})$$

$$v_i = 2.20 \text{ m/s}$$

Total momentum is conserved as I push the earth down and myself up:

$$0 = -(5.98 \times 10^{24} \text{ kg})v_e + (85.0 \text{ kg})(2.20 \text{ m/s})$$

$$v_e = \boxed{\sim 10^{-23} \text{ m/s}}$$

9.6 (a) For the system of two blocks $\Delta p = 0$,

or $p_i = p_f$

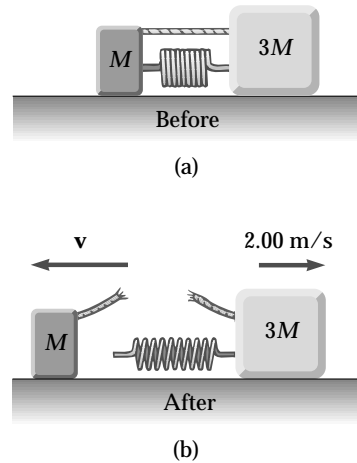
Therefore,

$$0 = Mv_m + (3M)(2.00 \text{ m/s})$$

Solving gives

$$v_m = \boxed{-6.00 \text{ m/s}} \text{ (motion toward the left)}$$

(b) $\frac{1}{2} kx^2 = \frac{1}{2} Mv_M^2 + \frac{1}{2} (3M) v_{3M}^2 = \boxed{8.40 \text{ J}}$



***9.7** (a) The momentum is $p = mv$, so $v = p/m$ and

the kinetic energy is $K = \frac{1}{2} mv^2 = \frac{1}{2} m \left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$

(b) $K = \frac{1}{2} mv^2$ implies $v = \sqrt{2K/m}$, so

$$p = mv = m\sqrt{2K/m} = \boxed{\sqrt{2mK}}$$

9.8 $I = \Delta p = m \Delta v = (70.0 \text{ kg})(5.20 \text{ m/s}) = \boxed{364 \text{ kg} \cdot \text{m/s}}$

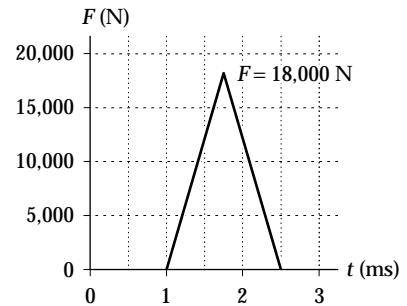
$$F = \frac{\Delta p}{\Delta t} = \frac{364}{0.832} = \boxed{438 \text{ N}}$$

9.9 (a) $I = \int F dt = \text{area under curve}$

$$= \frac{1}{2} (1.50 \times 10^{-3} \text{ s})(18000 \text{ N}) = \boxed{13.5 \text{ N} \cdot \text{s}}$$

(b) $F = \frac{13.5 \text{ N} \cdot \text{s}}{1.50 \times 10^{-3} \text{ s}} = \boxed{9.00 \text{ kN}}$

(c) From the graph, we see that $F_{\text{max}} = \boxed{18.0 \text{ kN}}$



9.10 Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse,

$$\mathbf{I} = \Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = (0.0600)(40.0)\mathbf{i} - (0.0600)(50.0)(-\mathbf{i}) = \boxed{5.40\mathbf{i} \text{ N} \cdot \text{s}}$$

(b) $\text{Work} = K_f - K_i = \frac{1}{2} (0.0600) [(40.0)^2 - (50.0)^2] =$

$$\boxed{-27.0 \text{ J}}$$

9.11 $\Delta p = F \Delta t$

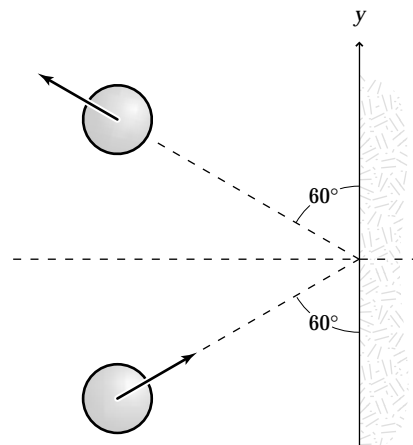
$$\Delta p_y = m(v_{fy} - v_{iy}) = m(v \cos 60.0^\circ) - mv \cos 60.0^\circ = 0$$

$$\Delta p_x = m(-v \sin 60.0^\circ - v \sin 60.0^\circ) = -2mv \sin 60.0^\circ$$

$$= -2(3.00 \text{ kg})(10.0 \text{ m/s})(0.866)$$

$$= -52.0 \text{ kg} \cdot \text{m/s}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0 \text{ kg} \cdot \text{m/s}}{0.200 \text{ s}} = \boxed{-260 \text{ N}}$$



Goal Solution

G: If we think about the angle as a variable and consider the limiting cases, then the force should be zero when the angle is 0° (no contact between the ball and the wall). When the angle is 90° the force will be its maximum and can be found from the momentum-impulse equation, so that $F < 300\text{N}$, and the force on the ball must be directed to the left.

O: Use the momentum-impulse equation to find the force, and carefully consider the direction of the velocity vectors by defining up and to the right as positive.

$$\mathbf{A:} \quad \Delta \mathbf{p} = \mathbf{F} \Delta t$$

$$\Delta p_y = mv_{yf} - mv_{yx} = m(v \cos 60.0^\circ - v \cos 60.0^\circ) = 0$$

So the wall does not exert a force on the ball in the y direction.

$$\Delta p_x = mv_{xf} - mv_{xx} = m(-v \sin 60.0^\circ - v \sin 60.0^\circ) = -2mv \sin 60.0^\circ$$

$$\Delta p_x = -2(3.00 \text{ kg})(10.0 \text{ m/s})(0.866) = -52.0 \text{ kg} \cdot \text{m/s}$$

$$\bar{\mathbf{F}} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{\Delta p_x \mathbf{i}}{\Delta t} = \frac{-52.0 \text{ kg} \cdot \text{m/s}}{0.200 \text{ s}} = -260 \text{ N} \mathbf{i}$$

$\mathbf{L:}$ The force is to the left and has a magnitude less than 300 N as expected.

9.12 Take x -axis toward the pitcher

$$(a) \quad p_{ix} + I_x = p_{fx}$$

$$0.200 \text{ kg}(15.0 \text{ m/s})(-\cos 45.0^\circ) + I_x = 0.200 \text{ kg}(40.0 \text{ m/s}) \cos 30.0^\circ$$

$$I_x = 9.05 \text{ N} \cdot \text{s}$$

$$p_{iy} + I_y = p_{fy}$$

$$0.200 \text{ kg}(15.0 \text{ m/s})(-\sin 45.0^\circ) + I_y = 0.200 \text{ kg}(40.0 \text{ m/s}) \sin 30.0^\circ$$

$$\mathbf{I} = \boxed{(9.05\mathbf{i} + 6.12\mathbf{j}) \text{ N} \cdot \text{s}}$$

$$(b) \quad \mathbf{I} = \frac{1}{2}(\mathbf{0} + \mathbf{F}_m)(4.00 \text{ ms}) + \mathbf{F}_m(20.0 \text{ ms}) + \frac{1}{2}\mathbf{F}_m(4.00 \text{ ms})$$

$$\mathbf{F}_m \times 24.0 \times 10^{-3} \text{ s} = (9.05\mathbf{i} + 6.12\mathbf{j}) \text{ N} \cdot \text{s}$$

$$\mathbf{F}_m = \boxed{(377\mathbf{i} + 255\mathbf{j}) \text{ N}}$$

9.13 The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600 \text{ kg})(25.0 \text{ m/s}) - 0}{1.00 \text{ s}} = \boxed{15.0 \text{ N}}$$

According to Newton's 3rd law, the water exerts a force of equal magnitude back on the hose. Thus, the holder must apply a 15.0 N force (in the direction of the velocity of the exiting water steam) to hold the hose stationary.

- *9.14** If the diver starts from rest and drops vertically into the water, the velocity just before impact is found from

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2} m v_{\text{impact}}^2 + 0 = 0 + mgh \Rightarrow v_{\text{impact}} = \sqrt{2gh}$$

With the diver at rest after an impact time of Δt , the average force during impact is given by

$$\bar{F} = \frac{m(0 - v_{\text{impact}})}{\Delta t} = \frac{-m\sqrt{2gh}}{\Delta t} \quad \text{or} \quad \bar{F} = \frac{m\sqrt{2gh}}{\Delta t} \quad (\text{directed upward})$$

Assuming a mass of 55 kg and an impact time of ≈ 1.0 s, the magnitude of this average force is

$$\left| \bar{F} \right| = \frac{(55 \text{ kg}) \sqrt{2(9.8 \text{ m/s}^2)(10 \text{ m})}}{1.0 \text{ s}} = 770 \text{ N, or } \boxed{\sim 10^3 \text{ N}}$$

- *9.15** $(200 \text{ g})(55.0 \text{ m/s}) = (46.0 \text{ g})v + (200 \text{ g})(40.0 \text{ m/s})$

$$v = \boxed{65.2 \text{ m/s}}$$

- 9.16** For each skater,

$$\bar{F} = \frac{m \Delta v}{\Delta t} = \frac{(75.0 \text{ kg})(5.00 \text{ m/s})}{(0.100 \text{ s})} = \boxed{3750 \text{ N}}$$

Since $\bar{F} < 4500 \text{ N}$, there are no broken bones.

- 9.17** Momentum is conserved

$$(10.0 \times 10^{-3} \text{ kg})v = (5.01 \text{ kg})(0.600 \text{ m/s})$$

$$v = \boxed{301 \text{ m/s}}$$

Goal Solution

G: A reasonable speed of a bullet should be somewhere between 100 and 1000 m/s.

O: We can find the initial speed of the bullet from conservation of momentum. We are told that the block of wood was originally stationary.

A: Since there is no external force on the block and bullet system, the total momentum of the system is constant so that $\Delta\mathbf{p} = 0$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$(0.0100 \text{ kg})v_{1i} + 0 = (0.0100 \text{ kg})(0.600 \text{ m/s})\mathbf{i} + (5.00 \text{ kg})(0.600 \text{ m/s}) \mathbf{i}$$

$$v_{1i} = \frac{(5.01 \text{ kg})(0.600 \text{ m/s})\mathbf{i}}{0.0100 \text{ kg}} = 301 \mathbf{i} \text{ m/s}$$

L: The speed seems reasonable, and is in fact just under the speed of sound in air (343 m/s at 20°C).

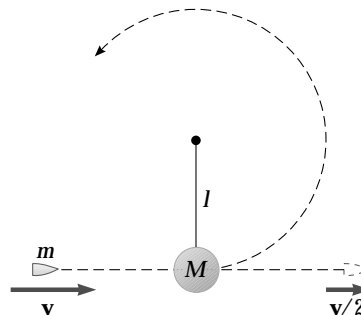
9.18 Energy is conserved for the bob between bottom and top of swing:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} Mv_b^2 + 0 = 0 + Mg2l$$

$$v_b^2 = g4l$$

$$v_b = 2\sqrt{gl}$$



Momentum is conserved in the collision:

$$mv = m\frac{v}{2} + M \cdot 2\sqrt{gl}$$

$$v = \frac{4M}{m} \sqrt{gl}$$

9.19 (a) and (b) Let v_g and v_p be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that

$$v_g - v_p = 1.50 \quad (1)$$

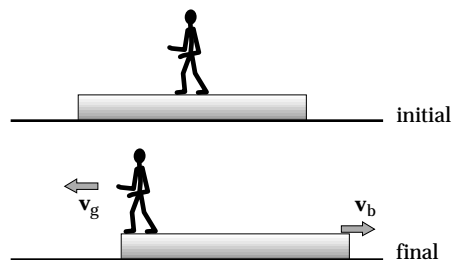
But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0v_g + 150v_p = 0, \quad \text{or} \quad v_g = -3.33 v_p$$

Putting this into the equation (1) above gives

$$-3.33 v_p - v_p = 1.50, \quad \text{or} \quad v_p = \boxed{-0.346 \text{ m/s}}$$

$$\text{Then } v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$$



9.20 Gayle jumps on the sled:

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$(50.0 \text{ kg})(4.00 \text{ m/s}) = (50.0 \text{ kg} + 5.00 \text{ kg})v_2$$

$$v_2 = 3.64 \text{ m/s}$$

They glide down 5.00 m:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} (55.0 \text{ kg})(3.64 \text{ m/s})^2 + 55.0 \text{ kg}(9.8 \text{ m/s}^2)5.00 \text{ m} = \frac{1}{2} (55.0 \text{ kg}) v_3^2$$

$$v_3 = 10.5 \text{ m/s}$$

Brother jumps on:

$$55.0 \text{ kg}(10.5 \text{ m/s}) + 0 = (85.0 \text{ kg})v_4$$

$$v_4 = 6.82 \text{ m/s}$$

All slide 10.0 m down:

$$\frac{1}{2} (85.0 \text{ kg})(6.82 \text{ m/s})^2 + 85.0 \text{ kg} (9.80 \text{ m/s}^2)10.0 \text{ m} = \frac{1}{2} (85.0 \text{ kg}) v_5^2$$

$$v_5 = \boxed{15.6 \text{ m/s}}$$

9.21 $p_i = p_f$

$$(a) \quad m_c v_{ic} + m_T v_{iT} = m_c v_{fc} + m_T v_{fT}$$

$$v_{fT} = \left(\frac{1}{m_T} \right) [m_c v_{ic} + m_T v_{iT} - m_c v_{fc}]$$

$$v_{fT} = \left(\frac{1}{9000 \text{ kg}} \right) [(1200 \text{ kg})(25.0 \text{ m/s}) + (9000 \text{ kg})(20.0 \text{ m/s}) - (1200 \text{ kg})(18.0 \text{ m/s})]$$

$$= \boxed{20.9 \text{ m/s}} \quad \text{East}$$

$$(b) \quad K_{\text{lost}} = K_i - K_f$$

$$= \frac{1}{2} m_c v_{ic}^2 + \frac{1}{2} m_T v_{iT}^2 - \frac{1}{2} m_c v_{fc}^2 - \frac{1}{2} m_T v_{fT}^2$$

$$= \frac{1}{2} [m_c (v_{ic}^2 - v_{fc}^2) + m_T (v_{iT}^2 - v_{fT}^2)]$$

$$= \frac{1}{2} [(1200 \text{ kg})(625 - 324)(\text{m}^2/\text{s}^2) + (9000 \text{ kg})(400 - 438.2)\text{m}^2/\text{s}^2]$$

$$K_{\text{lost}} = \boxed{8.68 \text{ kJ}} \quad \text{becomes internal energy.}$$

(If 20.9 m/s were used to determine the energy lost instead of 20.9333, the answer would be very different. We keep extra significant figures until the problem is complete!)

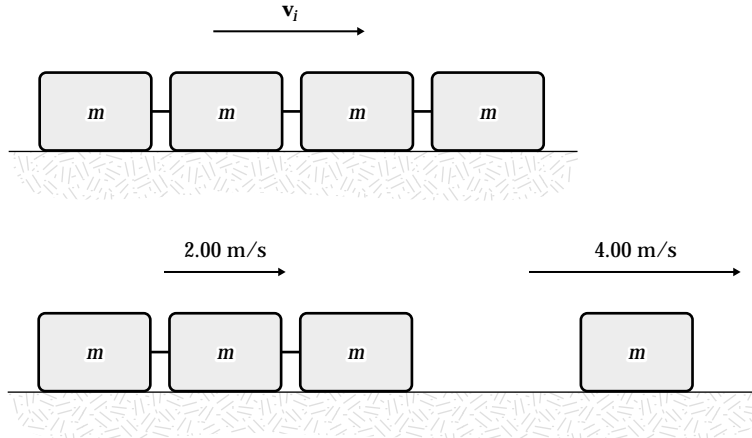
9.22 (a) $mv_{1i} + 3mv_{2i} = 4mv_f$ where $m = 2.50 \times 10^4 \text{ kg}$

$$v_f = \frac{4.00 + 3(2.00)}{4} = \boxed{2.50 \text{ m/s}}$$

$$(b) \quad K_f - K_i = \frac{1}{2} (4m) v_f^2 - \left[\frac{1}{2} m v_{1i}^2 + \frac{1}{2} (3m) v_{2i}^2 \right]$$

$$= 2.50 \times 10^4 [12.5 - 8.00 - 6.00] = \boxed{-3.75 \times 10^4 \text{ J}}$$

- *9.23** (a) The internal forces exerted by the actor do not change total momentum.



$$(4m)v_i = (3m)(2.00 \text{ m/s}) + m(4.00 \text{ m/s})$$

$$v_i = \frac{6.00 \text{ m/s} + 4.00 \text{ m/s}}{4} = \boxed{2.50 \text{ m/s}}$$

(b) $W_{\text{actor}} = K_f - K_i = \frac{1}{2} [(3m)(2.00 \text{ m/s})^2 + m(4.00 \text{ m/s})^2] - \frac{1}{2} (4m)(2.50 \text{ m/s})^2$

$$W_{\text{actor}} = \frac{(2.50 \times 10^4 \text{ kg})}{2} [12.0 + 16.0 - 25.0](\text{m/s})^2 = \boxed{37.5 \text{ kJ}}$$

- (c) The explosion considered here is the time reversal of the perfectly inelastic collision in problem 9.22. The same momentum conservation equation describes both processes.

- *9.24** We call the initial speed of the bowling ball v_i and from momentum conservation,

$$(7.00 \text{ kg})(v_i) + (2.00 \text{ kg})(0) = (7.00 \text{ kg})(1.80 \text{ m/s}) + (2.00 \text{ kg})(3.00 \text{ m/s})$$

gives

$$v_i = \boxed{2.66 \text{ m/s}}$$

- 9.25** (a) Following Example 9.8, the fraction of total kinetic energy transferred to the moderator is

$$f_2 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

where m_2 is the moderator nucleus and in this case,

$$m_2 = 12m_1$$

$$f_2 = \frac{4m_1(12m_1)}{(13m_1)^2} = \frac{48}{169} = \boxed{0.284 \text{ or } 28.4\%}$$

of the neutron energy is transferred to the carbon nucleus.

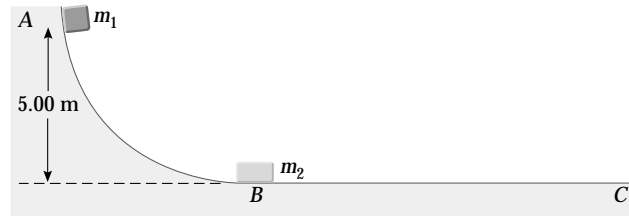
$$(b) \quad K_C = (0.284)(1.6 \times 10^{-13} \text{ J}) = \boxed{4.54 \times 10^{-14} \text{ J}}$$

$$K_D = (0.716)(1.6 \times 10^{-13} \text{ J}) = \boxed{1.15 \times 10^{-13} \text{ J}}$$

9.26 v_1 , speed of m_1 at B before collision.

$$\frac{1}{2} m_1 v_1^2 = m_1 g h$$

$$v_1 = \sqrt{2 \times 9.80 \times 5.00} = 9.90 \text{ m/s}$$



v_{1f} , speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_1 = -\frac{1}{3}(9.90) \text{ m/s} = -3.30 \text{ m/s}$$

At the highest point (after collision)

$$m_1 g h_{\max} = \frac{1}{2} m_1 (-3.30)^2$$

$$h_{\max} = \frac{(-3.30 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.556 \text{ m}}$$

9.27 At impact momentum is conserved, so:

$$m_1 v_1 = (m_1 + m_2) v_2$$

After impact the change in kinetic energy is equal to the work done by friction:

$$\frac{1}{2} (m_1 + m_2) v_2^2 = f_f d = \mu (m_1 + m_2) g d$$

$$\frac{1}{2} (0.112 \text{ kg}) v_2^2 = 0.650(0.112 \text{ kg})(9.80 \text{ m/s}^2)(7.50 \text{ m})$$

$$v_2^2 = 95.6 \text{ m}^2/\text{s}^2$$

$$v_2 = 9.77 \text{ m/s}$$

$$(12.0 \times 10^{-3} \text{ kg}) v_1 = (0.112 \text{ kg})(9.77 \text{ m/s})$$

$$v_1 = \boxed{91.2 \text{ m/s}}$$

- 9.28** We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These equal forces act backwards on the two bullets.

For the first, $K_i + \Delta E = K_f$

$$\frac{1}{2}(7.00 \times 10^{-3} \text{ kg}) v^2 + F(8.00 \times 10^{-2} \text{ m}) \cos 180^\circ = 0$$

For the second, $p_i = p_f$

$$(7.00 \times 10^{-3} \text{ kg})v = (1.014 \text{ kg})v_f$$

$$v_f = \frac{(7.00 \times 10^{-3})v}{1.014}$$

Again, $K_i + \Delta E = K_f$

$$\frac{1}{2}(7.00 \times 10^{-3} \text{ kg}) v^2 + Fd \cos 180^\circ = \frac{1}{2}(1.014 \text{ kg}) v_f^2$$

Substituting,

$$\frac{1}{2}(7.00 \times 10^{-3} \text{ kg}) v^2 - Fd = \frac{1}{2}(1.014 \text{ kg}) \left(\frac{7.00 \times 10^{-3} v}{1.014} \right)^2$$

$$Fd = \frac{1}{2}(7.00 \times 10^{-3} \text{ kg}) v^2 - \frac{1}{2}(7.00 \times 10^{-3} \text{ kg}) \frac{7.00 \times 10^{-3}}{1.014} v^2$$

Substituting again,

$$Fd = F(8.00 \times 10^{-2} \text{ m}) \left(1 - \frac{7.00 \times 10^{-3}}{1.014} \right)$$

$$d = \boxed{7.94 \text{ cm}}$$

- *9.29** (a) First, we conserve momentum in the x direction (the direction of travel of the fullback).

$$(90.0 \text{ kg})(5.00 \text{ m/s}) + 0 = (185 \text{ kg})V \cos \theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V \cos \theta = 2.43 \text{ m/s} \quad (1)$$

Now consider conservation of momentum in the y direction (the direction of travel of the opponent).

$$(95.0 \text{ kg})(3.00 \text{ m/s}) + 0 = (185 \text{ kg})(V \sin \theta)$$

which gives,

$$V \sin \theta = 1.54 \text{ m/s} \quad (2)$$

Divide equation (2) by (1)

$$\tan \theta = \frac{1.54}{2.43} = 0.633$$

From which

$$\theta = 32.3^\circ$$

Then, either (1) or (2) gives

$$V = 2.88 \text{ m/s}$$

$$(b) \quad K_i = \frac{1}{2} (90.0 \text{ kg})(5.00 \text{ m/s})^2 + \frac{1}{2} (95.0 \text{ kg})(3.00 \text{ m/s})^2 = 1.55 \times 10^3 \text{ J}$$

$$K_f = \frac{1}{2} (185 \text{ kg})(2.88 \text{ m/s})^2 = 7.67 \times 10^2 \text{ J}$$

Thus, the kinetic energy lost is 783 J into internal energy.

***9.30** The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0 \text{ m/s})$$

and $v_{Bi} = 8.33 \text{ m/s}$

$$K_i = \frac{1}{2} m(10.0 \text{ m/s})^2 + \frac{1}{2} (1.20m)(8.33 \text{ m/s})^2 = \frac{1}{2} m(183 \text{ m}^2/\text{s}^2)$$

$$K_f = \frac{1}{2} m(v_G)^2 + \frac{1}{2} (1.20m)(v_B)^2 = \frac{1}{2} \left(\frac{1}{2} m(183 \text{ m}^2/\text{s}^2) \right)$$

$$\text{or} \quad v_G^2 + 1.20v_B^2 = 91.7 \text{ m}^2/\text{s}^2 \quad (1)$$

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

$$\text{or} \quad v_G = 1.20v_B \quad (2)$$

Solving (1) and (2) simultaneously, we find

$$v_G = 7.07 \text{ m/s} \quad (\text{speed of green puck after collision})$$

$$\text{and} \quad v_B = 5.89 \text{ m/s} \quad (\text{speed of blue puck after collision})$$

***9.31** We use conservation of momentum for both northward and eastward components.

$$\text{For the eastward direction: } M(13.0 \text{ m/s}) = 2MV_f \cos 55.0^\circ$$

$$\text{For the northward direction: } MV = 2MV_f \sin 55.0^\circ$$

Divide the northward equation by the eastward equation to find:

$$V = (13.0 \text{ m/s}) \tan 55.0^\circ = 18.6 \text{ m/s} = \boxed{41.5 \text{ mi/h}}$$

Thus, the driver of the north bound car was untruthful.

***9.32** (a) $\mathbf{p}_i = \mathbf{p}_f$

$$\text{so } p_{xi} = p_{xf}$$

$$\text{and } p_{yi} = p_{yf}$$

$$mv_i = mv \cos \theta + mv \cos \phi \quad (1)$$

$$0 = mv \sin \theta - mv \sin \phi \quad (2)$$

From (2),

$$\sin \theta = \sin \phi \text{ so } \theta = \phi$$

Furthermore, energy conservation requires

$$\frac{1}{2} mv_i^2 = \frac{1}{2} mv^2 + \frac{1}{2} mv^2$$

$$\text{so } \boxed{v = \frac{v_i}{\sqrt{2}}}$$

(b) Hence, (1) gives

$$v_i = \frac{2v_i \cos \theta}{\sqrt{2}}$$

$$\theta = \boxed{45.0^\circ} \quad \phi = \boxed{45.0^\circ}$$

9.33 By conservation of momentum (with all masses equal),

$$5.00 \text{ m/s} + 0 = (4.33 \text{ m/s}) \cos 30.0^\circ + v_{2fx}$$

$$v_{2fx} = 1.25 \text{ m/s}$$

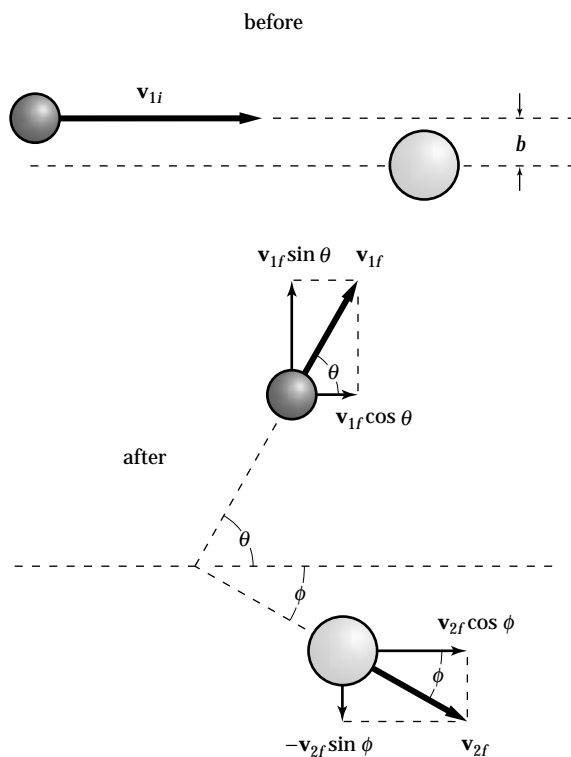
$$0 = (4.33 \text{ m/s}) \sin 30.0^\circ + v_{2fy}$$

$$v_{2fy} = -2.16 \text{ m/s}$$

$$\mathbf{v} = \boxed{2.50 \text{ m/s at } -60.0^\circ}$$

Note that we did not need to use the fact that the collision is perfectly elastic.

9.34 (a) Use Equations 9.24 and 9.25 and refer to the figures below.



Let the puck initially at rest be m_2 .

$$m_1 v_{1i} = m_1 v_{1f} \cos \theta + m_2 v_{2f} \cos \phi$$

$$0 = m_1 v_{1f} \sin \theta - m_2 v_{2f} \sin \phi$$

$$(0.200 \text{ kg})(2.00 \text{ m/s}) = (0.200 \text{ kg})(1.00 \text{ m/s}) \cos 53.0^\circ + (0.300 \text{ kg})v_{2f} \cos \phi$$

$$0 = (0.200 \text{ kg})(1.00 \text{ m/s}) \sin 53.0^\circ - (0.300 \text{ kg})(v_{2f} \sin \phi)$$

From these equations we find

$$\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{0.160}{0.280} = 0.571, \quad \phi = 29.7^\circ$$

Then

$$v_{2f} = \frac{(0.160 \text{ kg} \cdot \text{m/s})}{(0.300 \text{ kg})(\sin 29.7^\circ)} = 1.07 \text{ m/s}$$

(b) $f_{\text{lost}} = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = -0.318$

9.35 $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

$$3.00(5.00)\mathbf{i} - 6.00\mathbf{j} = 5.00\mathbf{v}$$

$$\mathbf{v} = (3.00\mathbf{i} - 1.20\mathbf{j}) \text{ m/s}$$

9.36 $p_{xf} = p_{xi}$

$$mv_O \cos 37.0^\circ + mv_Y \cos 53.0^\circ = m(5.00 \text{ m/s})$$

$$0.799v_O + 0.602v_Y = 5.00 \text{ m/s} \quad (1)$$

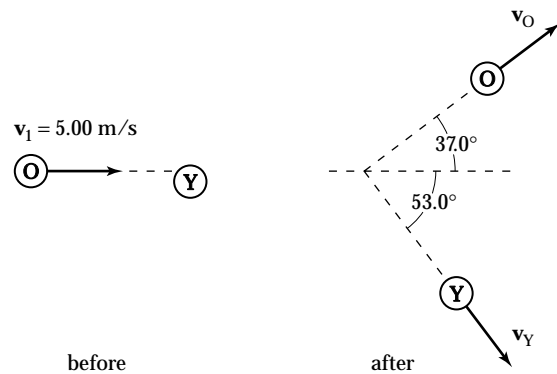
$$p_{yf} = p_{yi}$$

$$mv_O \sin 37.0^\circ - mv_Y \sin 53.0^\circ = 0$$

$$0.602v_O = 0.799v_Y \quad (2)$$

Solving (1) and (2) simultaneously,

$$v_O = 3.99 \text{ m/s} \quad \text{and} \quad v_Y = 3.01 \text{ m/s}$$



9.37 $p_{xf} = p_{xi}$

$$mv_O \cos \theta + mv_Y \cos (90.0^\circ - \theta) = mv_i$$

$$v_O \cos \theta + v_Y \sin \theta = v_i \quad (1)$$

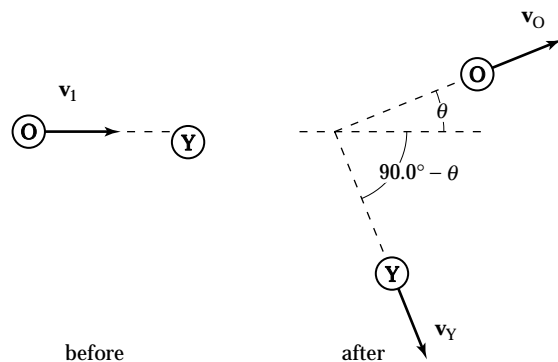
$$p_{yf} = p_{yi}$$

$$mv_O \sin \theta - mv_Y \sin (90.0^\circ - \theta) = 0$$

$$v_O \sin \theta = v_Y \cos \theta \quad (2)$$

From equation (2),

$$v_O = v_Y (\cos \theta / \sin \theta) \quad (3)$$



Substituting into equation (1),

$$v_Y \left(\frac{\cos^2 \theta}{\sin \theta} \right) + v_Y \sin \theta = v_i$$

so $v_Y(\cos^2 \theta + \sin^2 \theta) = v_i \sin \theta$, and $v_Y = v_i \sin \theta$

Then, from equation (3), $v_O = v_i \cos \theta$

9.38 The horizontal and vertical components of momentum are conserved:

$$(5.00 \text{ g})(250 \text{ m/s}) \cos 20.0^\circ - (3.00 \text{ g})(280 \text{ m/s}) \cos 15.0^\circ = (8.00 \text{ g})v_{fx}$$

$$v_{fx} = 45.4 \text{ m/s}$$

$$(5.00 \text{ g})(250 \text{ m/s}) \sin 20.0^\circ + (3.00 \text{ g})(280 \text{ m/s}) \sin 15.0^\circ = (8.00 \text{ g})v_{fy}$$

$$v_{fy} = 80.6 \text{ m/s}$$

$$\mathbf{v} = \boxed{45.4 \text{ m/s } \mathbf{i} + 80.6 \text{ m/s } \mathbf{j}} = 92.5 \text{ m/s at } 60.6^\circ$$

9.39 $m_0 = 17.0 \times 10^{-27} \text{ kg}$ $\mathbf{v}_i = 0$ (the parent nucleus)

$$m_1 = 5.00 \times 10^{-27} \text{ kg} \quad \mathbf{v}_1 = 6.00 \times 10^6 \text{ j m/s}$$

$$m_2 = 8.40 \times 10^{-27} \text{ kg} \quad \mathbf{v}_2 = 4.00 \times 10^6 \text{ i m/s}$$

(a) $m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 + m_3 \mathbf{v}_3 = 0$

where $m_3 = m_0 - m_1 - m_2 = 3.60 \times 10^{-27} \text{ kg}$

$$(5.00 \times 10^{-27})(6.00 \times 10^6 \text{ j}) + (8.40 \times 10^{-27})(4.00 \times 10^6 \text{ i}) + (3.60 \times 10^{-27})\mathbf{v}_3 = 0$$

$$\mathbf{v}_3 = \boxed{(-9.33 \times 10^6 \text{ i} - 8.33 \times 10^6 \text{ j}) \text{ m/s}}$$

(b) $E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2$

$$= \frac{1}{2} [(5.00 \times 10^{-27})(6.00 \times 10^6)^2 + (8.40 \times 10^{-27})(4.00 \times 10^6)^2$$

$$+ (3.60 \times 10^{-27})(12.5 \times 10^6)^2]$$

$$\boxed{E = 4.39 \times 10^{-13} \text{ J}}$$

***9.40** The x -coordinate of the center of mass is

$$x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(0 + 0 + 0 + 0)}{(2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg})}$$

$$\boxed{x_{\text{CM}} = 0}$$

and the y -coordinate of the center of mass is

$$y_{\text{CM}} = \frac{\sum m_i y_i}{\sum m_i}$$

$$y_{\text{CM}} = \frac{(2.00 \text{ kg})(3.00 \text{ m}) + (3.00 \text{ kg})(2.50 \text{ m}) + (2.50 \text{ kg})(0) + (4.00 \text{ kg})(-0.500 \text{ m})}{2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg}}$$

$$\boxed{y_{\text{CM}} = 1.00 \text{ m}}$$

9.41 Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.

Goal Solution

G: By inspection, it appears that the center of mass is located at about $(12 \mathbf{i} + 13 \mathbf{j})$ cm.

O: Think of the sheet as composed of three sections, and consider the mass of each section to be at the geometric center of that section. Define the mass per unit area to be σ , and number the rectangles as shown. We can then calculate the mass and identify the center of mass of each section.

A: $m_{\text{I}} = (30.0 \text{ cm})(10.0 \text{ cm})\sigma$ $\text{CM}_{\text{I}} = (15.0 \text{ cm}, 5.0 \text{ cm})$

$m_{\text{II}} = (10.0 \text{ cm})(10.0 \text{ cm})\sigma$ $\text{CM}_{\text{II}} = (5.0 \text{ cm}, 15.0 \text{ cm})$

$m_{\text{III}} = (10.0 \text{ cm})(20.0 \text{ cm})\sigma$ $\text{CM}_{\text{III}} = (10.0 \text{ cm}, 25.0 \text{ cm})$

The overall CM is at a point defined by the vector equation $\mathbf{r}_{\text{cm}} \equiv \frac{\sum m_i \mathbf{r}_i}{\sum m_i}$

Substituting the appropriate values, \mathbf{r}_{CM} is calculated to be:

$$\mathbf{r}_{\text{CM}} = \frac{(300\sigma \text{ cm}^3)(15.0\mathbf{i} + 5.0\mathbf{j}) + (100\sigma \text{ cm}^3)(5.0\mathbf{i} + 15.0\mathbf{j}) + (200\sigma \text{ cm}^3)(10.0\mathbf{i} + 25.0\mathbf{j})}{(300 + 200 + 100)\sigma \text{ cm}^2}$$

$$\mathbf{r}_{\text{CM}} = \frac{(45.0\mathbf{i} + 15.0\mathbf{j} + 5.0\mathbf{i} + 15.0\mathbf{j} + 20.0\mathbf{i} + 50.0\mathbf{j})}{6.00} \text{ cm}$$

$$\mathbf{r}_{\text{CM}} = (11.7\mathbf{i} + 13.3\mathbf{j}) \text{ cm}$$

L: The coordinates are close to our eyeball estimate. In solving this problem, we could have chosen to divide the original shape some other way, but the answer would be the same. This problem also shows that the center of mass can lie outside the boundary of the object.

$$A = A_1 + A_2 + A_3$$

$$M = M_1 + M_2 + M_3$$

$$\frac{M_1}{A_1} = \frac{M}{A}$$

$$A_1 = 300 \text{ cm}^2, \quad A_2 = 100 \text{ cm}^2, \quad A_3 = 200 \text{ cm}^2$$

$$A = 600 \text{ cm}^2$$

$$M_1 = M \left(\frac{A_1}{A} \right) = \frac{300 \text{ cm}^2}{600 \text{ cm}^2} M = \frac{M}{2}$$

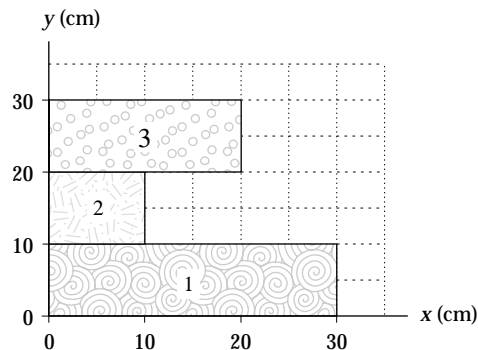
$$M_2 = M \left(\frac{A_2}{A} \right) = \frac{100 \text{ cm}^2}{600 \text{ cm}^2} M = \frac{M}{6}$$

$$M_3 = M \left(\frac{A_3}{A} \right) = \frac{200 \text{ cm}^2}{600 \text{ cm}^2} M = \frac{M}{3}$$

$$x_{\text{CM}} = \frac{x_1 M_1 + x_2 M_2 + x_3 M_3}{M} = \frac{15.0 \text{ cm} \left(\frac{1}{2} M \right) + 5.00 \text{ cm} \left(\frac{1}{6} M \right) + 10.0 \text{ cm} \left(\frac{1}{3} M \right)}{M}$$

$$x_{\text{CM}} = 11.7 \text{ cm}$$

$$y_{\text{CM}} = \frac{\frac{1}{2} M (5.00 \text{ cm}) + \frac{1}{6} M (15.0 \text{ cm}) + \left(\frac{1}{3} M \right) (25.0 \text{ cm})}{M} = 13.3 \text{ cm}$$



*9.42 We use, with $x = 0$ at the center of the earth,

$$x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0 + (7.36 \times 10^{22} \text{ kg})(3.84 \times 10^8 \text{ m})}{5.98 \times 10^{24} \text{ kg} + 7.36 \times 10^{22} \text{ kg}} = 4.67 \times 10^6 \text{ m}$$

This is 0.732 (radius of the earth).

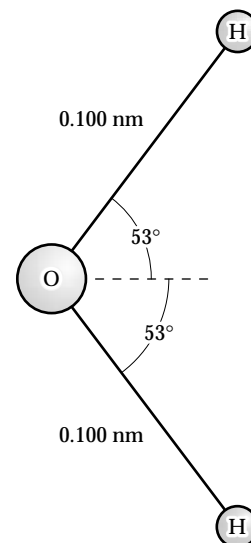
9.43 Take x -axis starting from the oxygen nucleus and pointing toward the middle of the V.

Then $y_{\text{CM}} = 0$

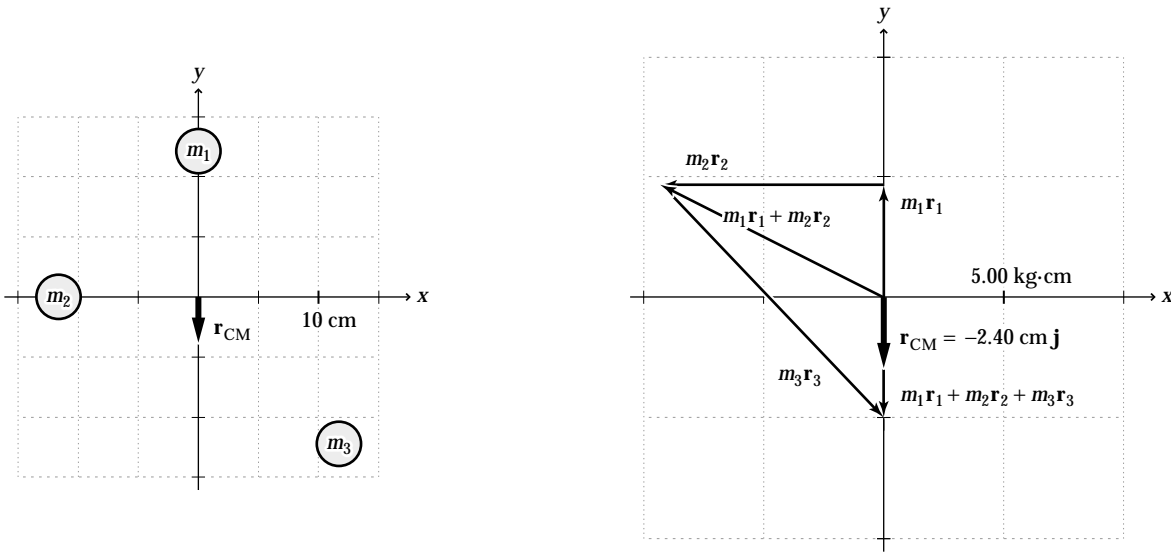
and $x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i}$

$$x_{\text{CM}} = \frac{0 + 1.008 \text{ u}(0.100 \text{ nm}) \cos 53.0^\circ + 1.008 \text{ u}(0.100 \text{ nm}) \cos 53.0^\circ}{(15.999 + 1.008 + 1.008) \text{ u}}$$

$$x_{\text{CM}} = 0.00673 \text{ nm from the oxygen nucleus}$$



*9.44



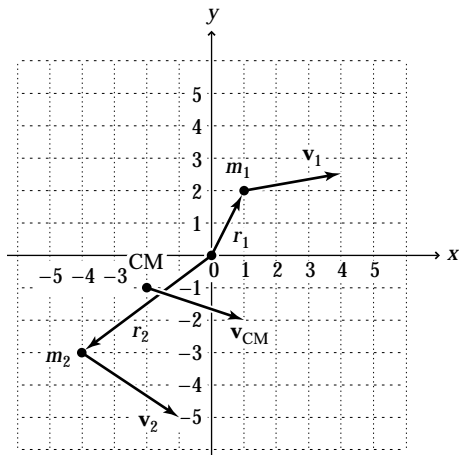
*9.45 (a) $M = \int dm = \int_0^{0.300 \text{ m}} \lambda dx = \int_0^{0.300 \text{ m}} [50.0 \text{ g/m} + 20.0x \text{ g/m}^2] dx$

$$M = [50.0x \text{ g/m} + 10.0x^2 \text{ g/m}^2]_0^{0.300 \text{ m}} = \boxed{15.9 \text{ g}}$$

(b) $x_{\text{CM}} = \frac{\int_{\text{all mass}} x dm}{M} = \frac{1}{M} \int_0^{0.300 \text{ m}} \lambda x dx = \frac{1}{M} \int_0^{0.300 \text{ m}} [50.0x \text{ g/m} + 20.0x^2 \text{ g/m}^2] dx$

$$x_{\text{CM}} = \frac{1}{15.9 \text{ g}} \left[25.0x^2 \text{ g/m} + \frac{20x^3 \text{ g/m}^2}{3} \right]_0^{0.300 \text{ m}} = \boxed{0.153 \text{ m}}$$

*9.46 (a)



(b) Using the definition of the position vector at the center of mass,

$$\mathbf{r}_{\text{CM}} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} = \frac{2.00 \text{ kg} \times (1.00 \text{ m}, 2.00 \text{ m}) + (3.00 \text{ kg})(-4.00 \text{ m}, -3.00 \text{ m})}{2.00 \text{ kg} + 3.00 \text{ kg}}$$

$$\mathbf{r}_{\text{CM}} = \boxed{(-2.00\mathbf{i} - 1.00\mathbf{j})\text{m}}$$

(c) The velocity of the center of mass is

$$\mathbf{v}_{\text{CM}} = \frac{\mathbf{P}}{M} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2}$$

$$= \frac{2.00 \text{ kg} \times (3.00 \text{ m/s}, 0.50 \text{ m/s}) + (3.00 \text{ kg})(3.00 \text{ m/s}, -2.00 \text{ m/s})}{(2.00 \text{ kg} + 3.00 \text{ kg})}$$

$$\mathbf{v}_{\text{CM}} = \boxed{(3.00\mathbf{i} - 1.00\mathbf{j})\text{m/s}}$$

(d) The total linear momentum of the system can be calculated as $\mathbf{P} = M\mathbf{v}_{\text{CM}}$ or as

$$\mathbf{P} = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2$$

Either gives

$$\mathbf{P} = \boxed{(15.0\mathbf{i} - 5.00\mathbf{j}) \text{ kg} \cdot \text{m/s}}$$

9.47 Let x = distance from shore to center of boat

l = length of boat

x' = distance boat moves as Juliet moves toward Romeo

$$\text{Before: } x_{\text{CM}} = \frac{\left[M_B x + M_J \left(x - \frac{l}{2} \right) + M_R \left(x + \frac{l}{2} \right) \right]}{(M_B + M_J + M_R)}$$

$$\text{After: } x_{\text{CM}} = \frac{\left[M_B (x - x') + M_J \left(x + \frac{l}{2} - x' \right) + M_R \left(x + \frac{l}{2} - x' \right) \right]}{(M_B + M_J + M_R)}$$

$$l \left(-\frac{55.0}{2} + \frac{77.0}{2} \right) = x' (-80.0 - 55.0 - 77.0) + \frac{l}{2} (55.0 + 77.0)$$

$$x' = \frac{55.0l}{212} = \frac{55.0(2.70)}{212} = \boxed{0.700 \text{ m}}$$

***9.48** $M\mathbf{r}_{\text{CM}} = m_1\mathbf{r}_1 + m_2\mathbf{r}_2$

(a) $M\mathbf{v}_{\text{CM}} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

$$(0.900 \text{ kg})\mathbf{v}_{\text{CM}} = (0.600 \text{ kg})[(-0.800 \text{ m/s}) \cos 45.0^\circ \mathbf{i} + (0.800 \text{ m/s}) \sin 45.0^\circ \mathbf{j}] \\ + (0.300 \text{ kg})[(0.800 \text{ m/s}) \cos 45.0^\circ \mathbf{i} + (0.800 \text{ m/s}) \sin 45.0^\circ \mathbf{j}]$$

$$\mathbf{v}_{\text{CM}} = \frac{-1.70 \text{ kg} \cdot \text{m/s} \mathbf{i} + 5.09 \text{ kg} \cdot \text{m/s} \mathbf{j}}{0.900 \text{ kg}} = \boxed{(-0.189\mathbf{i} + 0.566\mathbf{j}) \text{ m/s}}$$

(b) $v_{\text{CM}} = [(0.189 \text{ m/s})^2 + (0.566 \text{ m/s})^2]^{1/2}$ at $\tan^{-1}\left(\frac{0.566}{-0.189}\right)$

$$v_{\text{CM}} = \boxed{0.596 \text{ m/s at } \theta = 108^\circ}$$

(c) The center of mass starts from the origin at $t = 0$. At any later time, its location is

$$\mathbf{r}_{\text{CM}} = \mathbf{v}_{\text{CM}}t = \boxed{(-0.189\mathbf{i} + 0.566\mathbf{j})t \text{ m}}$$

9.49 (a) $\mathbf{v}_{\text{CM}} = \frac{\sum m_i \mathbf{v}_i}{M} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{M}$

$$\mathbf{v}_{\text{CM}} = \frac{(2.00 \text{ kg})(2.00\mathbf{i} - 3.00\mathbf{j}) \text{ m/s} + (3.00 \text{ kg})(1.00\mathbf{i} + 6.00\mathbf{j}) \text{ m/s}}{5.00 \text{ kg}}$$

$$\mathbf{v}_{\text{CM}} = \boxed{(1.40\mathbf{i} + 2.40\mathbf{j}) \text{ m/s}}$$

(b) $\mathbf{p} = M\mathbf{v}_{\text{CM}} = (5.00 \text{ kg})(1.40\mathbf{i} + 2.40\mathbf{j}) \text{ m/s} = \boxed{(7.00\mathbf{i} + 12.0\mathbf{j}) \text{ kg} \cdot \text{m/s}}$

***9.50** (a) Conservation of momentum:

$$2.00 \text{ kg}(1.50 \text{ m/s}) + 0.300 \text{ kg}(-0.400 \text{ m/s}) = 0.200 \text{ kg } v_{1f} + 0.300 \text{ kg } v_{2f}$$

Relative velocity equation:

$$v_{2f} - v_{1f} = 1.90 \text{ m/s}$$

Then

$$0.300 - 0.120 = 0.200v_{1f} + 0.300(1.90 + v_{1f})$$

$$\boxed{v_{1f} = -0.780 \text{ m/s}}$$

$$\boxed{v_{2f} = 1.12 \text{ m/s}}$$

(b) Before,

$$\mathbf{v}_{\text{CM}} = \frac{0.200 \text{ kg}(1.50 \text{ m/s})\mathbf{i} + 0.300 \text{ kg}(-0.400 \text{ m/s})\mathbf{i}}{0.500 \text{ kg}}$$

$$\mathbf{v}_{\text{CM}} = (0.360 \text{ m/s})\mathbf{i}$$

Afterwards, the center of mass must move at the same velocity, as momentum is conserved.

9.51 (a) Thrust = $\left| v_e \frac{dM}{dt} \right| = (2.60 \times 10^3 \text{ m/s})(1.50 \times 10^4 \text{ kg/s}) = 3.90 \times 10^7 \text{ N}$

(b) $\Sigma F_y = \text{Thrust} - Mg = Ma$

$$3.90 \times 10^7 - (3.00 \times 10^6)(9.80) = (3.00 \times 10^6)a$$

$$a = 3.20 \text{ m/s}^2$$

Goal Solution

G: The thrust must be at least equal to the weight of the rocket (30 MN); otherwise the launch will not be successful! However, since a Saturn V rocket accelerates rather slowly compared to the acceleration of falling objects, the thrust should be less than about twice the rocket's weight so that $0 < a < g$.

O: Use Newton's second law to find the force and acceleration from the changing momentum.

(a) The impulse due to the thrust, F , is equal to the change in momentum as fuel is exhausted from the rocket.

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv_e)$$

$$F = v_e \left(\frac{dm}{dt} \right)$$

Since v_e is a constant exhaust velocity, $\frac{dm}{dt}$ is the fuel consumption rate, so

$$F = (2.60 \times 10^3 \text{ m/s})(1.50 \times 10^4 \text{ kg/s}) = 39.0 \text{ MN}$$

(b) Applying $\Sigma F = ma$, $(3.90 \times 10^7 \text{ N}) - (3.00 \times 10^6 \text{ kg})(9.80 \text{ m/s}^2) = (3.00 \times 10^6 \text{ kg})a$

$$a = \frac{(3.90 \times 10^7 \text{ N}) - (29.4 \times 10^6 \text{ N})}{3.00 \times 10^6 \text{ kg}} = 3.20 \text{ m/s}^2 \text{ up}$$

L: As expected, the thrust is slightly greater than the weight of the rocket, and the acceleration is about 0.3 g, so the answers appear to be reasonable. This kind of rocket science is not so complicated after all!

- *9.52** (a) From equation 9.42, the thrust

$$T = 2.40 \times 10^7 \text{ N} = v_e \frac{dM}{dt}$$

If $v_e = 3000 \text{ m/s}$, then

$$\frac{dM}{dt} = \boxed{8000 \text{ kg/s}}$$

- (b) From Equation 9.41,

$$v_f - v_i = v_e \ln \left(\frac{M_i}{M_f} \right)$$

$$v_f - 0 = (3000 \text{ m/s}) \ln \left(\frac{M_i}{0.100M_i} \right)$$

$$v_f = \boxed{6.91 \text{ km/s}}$$

9.53 $v = v_r \ln \frac{M_i}{M_f}$

(a) $M_i = e^{v/v_r} M_f = e^5 (3.00 \times 10^3 \text{ kg}) = 4.45 \times 10^5 \text{ kg}$

The mass of fuel and oxidizer is

$$\Delta M = M_i - M_f = (445 - 3.00) \times 10^3 \text{ kg} = \boxed{442 \text{ metric tons}}$$

(b) $\Delta M = e^2 (3.00 \text{ metric tons}) - 3.00 \text{ metric tons} = \boxed{19.2 \text{ metric tons}}$

Because of the exponential, a relatively small increase in fuel and/or engine efficiency causes a large change in the amount of fuel and oxidizer required.

***9.54** (a) $v_f - 0 = v_e \ln \left(\frac{M_i}{M_f} \right)$

$$\ln \left(\frac{M_i}{M_f} \right) = \frac{v_f}{v_e}, \text{ so } \frac{M_i}{M_f} = \exp \left(\frac{v_f}{v_e} \right)$$

$$\text{or } M_f = M_i \exp \left(-\frac{v_f}{v_e} \right) = (5000 \text{ kg}) \exp \left(-\frac{225}{2500} \right) = 4.57 \times 10^3 \text{ kg}$$

$$M_{\text{fuel}} = M_i - M_f = \boxed{430 \text{ kg}}$$

(b) $430 \text{ kg} = (30.0 \text{ kg/s})t \Rightarrow t = \boxed{14.3 \text{ s}}$

9.55 (a) $(60.0 \text{ kg}) 4.00 \text{ m/s} = (120 + 60.0) \text{ kg } v_f$

$$v_f = \boxed{1.33 \text{ m/s } \mathbf{i}}$$



(b) $\Sigma F_y = 0$

$$n - (60.0 \text{ kg}) 9.80 \text{ m/s}^2 = 0$$

$$f_k = \mu_k n = 0.400 (588 \text{ N}) = \boxed{235 \text{ N}}$$

(c) For the person $p_i + I = p_f$

$$mv_i + Ft = mv_f$$

$$(60.0 \text{ kg}) 4.00 \text{ m/s} - (235 \text{ N})t = (60.0 \text{ kg}) 1.33 \text{ m/s}$$

$$t = \boxed{0.680 \text{ s}}$$

(d) person: $mv_f - mv_i = 60.0 \text{ kg} (1.33 - 4.00) \text{ m/s} = \boxed{-160 \text{ N} \cdot \text{s } \mathbf{i}}$

cart: $120 \text{ kg} (1.33 \text{ m/s}) - 0 = \boxed{+160 \text{ N} \cdot \text{s } \mathbf{i}}$

(e) $x - x_i = \frac{1}{2} (v_i + v) t$

$$= \frac{1}{2} [(4.00 + 1.33) \text{ m/s}] 0.680 \text{ s} = \boxed{1.81 \text{ m}}$$

(f) $x - x_i = \frac{1}{2} (v_i + v) t$

$$= \frac{1}{2} (0 + 1.33 \text{ m/s}) 0.680 \text{ s} = \boxed{0.454 \text{ m}}$$

(g) $\frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = \frac{1}{2} 60.0 \text{ kg} (1.33 \text{ m/s})^2 - 60.0 \text{ kg} (4.00 \text{ m/s})^2 = \boxed{-427 \text{ J}}$

(h) $\frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = \frac{1}{2} 120 \text{ kg} (1.33 \text{ m/s})^2 - 0 = \boxed{107 \text{ J}}$

- (i) Equal friction forces act through different distances on person and cart, to do different amounts of work on them. The total work on both together, -320 J , becomes $+320 \text{ J}$ of internal energy in this perfectly inelastic collision.