Chapter 15 Solutions

15.1 $M = \rho_{\text{iron}} V = (7860 \text{ kg/m}^3)\left[\frac{4}{3}\pi\,(0.0150 \text{ m})^3\right]$

$M = \boxed{0.111 \text{ kg}}$

15.2 The density of the nucleus is of the same order of magnitude as that of one proton, according to the assumption of close packing:

$$\rho = \frac{m}{V} \sim \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3}\pi (10^{-15} \text{ m})^3} \boxed{\sim 10^{18} \text{ kg/m}^3}$$

15.3 $P = \frac{F}{A} = \frac{50.0(9.80)}{\pi\,(0.500 \times 10^{-2})^2} = \boxed{6.24 \times 10^6 \text{ N/m}^2}$

***15.4** Let F_g be its weight. Then each tire supports $F_g/4$, so $P = \frac{F}{A} = \frac{F_g}{4A}$

yielding $F_g = 4AP = 4(0.0240 \text{ m}^2)(200 \times 10^3 \text{ N/m}^2) = \boxed{1.92 \times 10^4 \text{ N}}$

15.5 The Earth's surface area is $4\pi R^2$. The force pushing inward over this area amounts to

$$F = P_0 A = P_0 4\pi R^2$$

This force is the weight of the air:

$$F_g = mg = P_0 4\pi R^2$$

so the mass of the air is

$$m = \frac{P_0 4\pi R^2}{g} = \frac{(1.013 \times 10^5 \text{ N/m}^2)\, 4\pi (6.37 \times 10^6 \text{ m})^2}{9.80 \text{ m/s}^2} = \boxed{5.27 \times 10^{18} \text{ kg}}$$

15.6 (a) $P = P_0 + \rho g h = 1.013 \times 10^5 \text{ Pa} + (1024 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1000 \text{ m})$

$P = \boxed{1.01 \times 10^7 \text{ Pa}}$

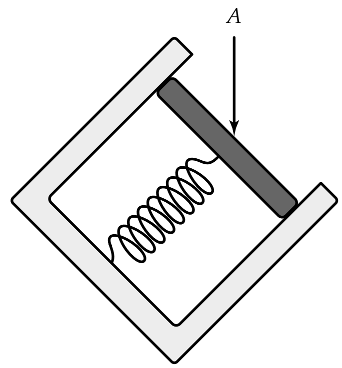
(b) The gauge pressure is the difference in pressure between the water outside and the air inside the submarine, which we suppose is at 1.00 atmosphere.

$$P_{\text{gauge}} = P - P_0 = \rho g h = 1.00 \times 10^7 \text{ Pa}$$

The resultant inward force on the porthole is then

$$F = P_{\text{gauge}} A = (1.00 \times 10^7 \text{ Pa})\,[\pi(0.150 \text{ m})^2] = \boxed{7.09 \times 10^5 \text{ N}}$$

15.7 $F_{el} = F_{fluid}$ or $kx = \rho g h A$



and $h = \dfrac{kx}{\rho g A}$

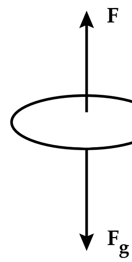
$$h = \frac{(1000\ \text{N/m}^2)(5.00 \times 10^{-3}\ \text{m})}{(10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\pi(1.00 \times 10^{-2}\ \text{m})^2} = \boxed{1.62\ \text{m}}$$

15.8 Since the pressure is the same on both sides, $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

In this case, $\dfrac{15\,000}{200} = \dfrac{F_2}{3.00}$

or $F_2 = \boxed{225\ \text{N}}$

15.9 $F_g = (80.0\ \text{kg})(9.80\ \text{m/s}^2) = 784\ \text{N}$



$F_g = F = PA = (1.013 \times 10^5\ \text{Pa})(A)$

$A = \dfrac{F_g}{P} = \dfrac{784}{1.013 \times 10^5} = \boxed{7.74 \times 10^{-3}\ \text{m}^2}$

Goal Solution

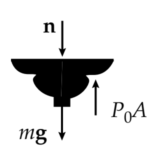
G: The suction cups used by burglars seen in movies are about 10 cm in diameter, and it seems reasonable that one of these might be able to support the weight of an 80-kg student. The area of a 10-cm cup is approximately: $\pi(0.05\ \text{m})^2 \cong 8 \times 10^{-3}\ \text{m}^2$

O: "Suction" is not a new kind of force. Familiar forces hold the cup in equilibrium, one of which is the atmospheric pressure acting over the area of the cup. This problem is simply one situation where Newton's 2nd law can be applied.

A: The vacuum between cup and ceiling exerts no force on either. The atmospheric pressure of the air below the cup pushes up on it with a force. If the cup barely supports the student's weight, then the normal force of the ceiling is approximately zero, and $+P_0A - mg = 0$

Therefore, $A = \dfrac{mg}{P_o} = \dfrac{80\ \text{kg}\ (9.8\ \text{m/s}^2)}{1.013 \times 10^5\ \text{Pa}} = 7.74 \times 10^{-3}\ \text{m}^2$ ◊

L: This calculated area agrees with our prediction and corresponds to a suction cup that is 9.93 cm in diameter (Our 10 cm estimate was right on—a lucky guess considering that a burglar would probably use at least two suction cups, not one.) Also, the suction cup in the drawing appears to be about 30 cm in diameter, plenty big enough to support the weight of the student.



***15.10** (a) Suppose the "vacuum cleaner" functions as a high–vacuum pump. The air below the brick will exert on it a lifting force

$$F = PA = (1.013 \times 10^3 \text{ Pa})[\pi(1.43 \times 10^{-2} \text{ m})^2] = \boxed{65.1 \text{ N}}$$

(b) The octopus can pull the bottom away from the top shell with a force that could be no larger than

$$F = PA = (P_0 + \rho g h)A$$

$$= [1.013 \times 10^3 \text{ Pa} + (1030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(32.3 \text{ m})][\pi(1.43 \times 10^{-2} \text{ m})^2]$$

$$F = \boxed{275 \text{ N}}$$

***15.11** The excess water pressure (over air pressure) halfway down is

$$P_{\text{gauge}} = \rho g h = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.20 \text{ m}) = 1.18 \times 10^4 \text{ Pa}$$

The force on the wall due to the water is

$$F = P_{\text{gauge}}A = (1.18 \times 10^4 \text{ Pa})(2.40 \text{ m})(9.60 \text{ m}) = \boxed{2.71 \times 10^5 \text{ N}}$$

horizontally toward the back of the hole.

15.12 The pressure on the bottom due to the water is

$$P_b = \rho g z = 1.96 \times 10^4 \text{ Pa}$$

So, $F_b = P_b A = \boxed{5.88 \times 10^6 \text{ N}}$

On each end,

$$F = \bar{P} A = (9.80 \times 10^3 \text{ Pa})(20.0 \text{ m}^2) = \boxed{196 \text{ kN}}$$

On the side

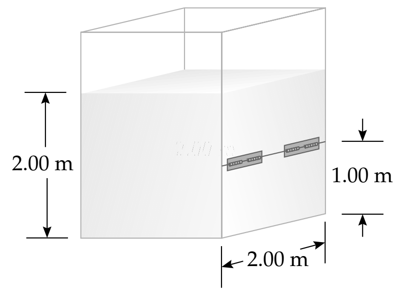
$$F = \bar{P} A = (9.80 \times 10^3 \text{ Pa})(60.0 \text{ m}^2) = \boxed{588 \text{ kN}}$$

***15.13** In the reference frame of the fluid, the cart's acceleration causes a fictitious force to act backward, as if the acceleration of gravity were $\sqrt{g^2 + a^2}$ directed downward and backward at $\theta = \tan^{-1}(a/g)$ from the vertical. The center of the spherical shell is at depth $d/2$ below the air bubble and the pressure there is

$$P = P_0 + \rho g_{\text{eff}} h = \boxed{P_0 + \frac{1}{2}\rho d\sqrt{g^2 + a^2}}$$

15.14 The air outside and water inside both exert atmospheric pressure, so only the excess water pressure ρgh counts for the net force. Take a strip of hatch between depth h and $h + dh$. It feels force

$$dF = PdA = \rho gh(2.00\ \text{m})dh$$



(a) The total force is

$$F = \int dF = \int_{h = 1.00\ \text{m}}^{2.00\ \text{m}} \rho gh\ (2.00\ \text{m})dh$$

$$F = \rho g(2.00\ \text{m})\ \frac{h^2}{2}\Bigg|_{1.00\ \text{m}}^{2.00\ \text{m}}$$

$$= (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\frac{(2.00\ \text{m})}{2}\ [(2.00\ \text{m})^2 - (1.00\ \text{m})^2]$$

$$F = \boxed{29.4\ \text{kN (to the right)}}$$

(b) The lever arm of dF is the distance $(h - 1.00\ \text{m})$ from hinge to strip:

$$\tau = \int d\tau = \int_{h = 1.00\ \text{m}}^{2.00\ \text{m}} \rho gh\ (2.00\ \text{m})(h - 1.00\ \text{m})dh$$

$$\tau = \rho g(2.00\ \text{m})\left[\frac{h^3}{3} - (1.00\ \text{m})\frac{h^2}{2}\right]_{1.00\ \text{m}}^{2.00\ \text{m}}$$

$$\tau = 1000\ \frac{\text{kg}}{\text{m}^3}(9.80\ \text{m/s}^2)(2.00\ \text{m})\left(\frac{7.00\ \text{m}^3}{3} - \frac{3.00\ \text{m}^3}{2}\right)$$

$$\tau = \boxed{16.3\ \text{kN}\cdot\text{m counterclockwise}}$$

15.15 The pressure on the ball is given by: $P = P_{atm} + \rho_w gh$ so the change in pressure on the ball from when it is on the surface of the ocean to when it is at the bottom of the ocean is $\Delta P = \rho_w gh$.

In addition:

$$\Delta V = \frac{-V\,\Delta P}{B} = -\frac{\rho_w\, ghV}{B} = -\frac{4\pi\rho_w ghr^3}{3B}$$, where B is the Bulk Modulus.

$$\Delta V = -\frac{4\pi(1030\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(10\ 000\ \text{m})(1.50\ \text{m})^3}{(3)(14.0 \times 10^{10}\ \text{Pa})} = -0.0102\ \text{m}^3$$

Therefore, the volume of the ball at the bottom of the ocean is

$$V - \Delta V = \frac{4}{3}\pi(1.50\ \text{m})^3 - 0.0102\ \text{m}^3 = 14.137\ \text{m}^3 - 0.0102\ \text{m}^3 = 14.127\ \text{m}^3$$

This gives a radius of 1.49964 m and a new diameter of 2.9993 m. Therefore the diameter decreases by $\boxed{0.722\ \text{mm}}$.

15.16 $\Delta P_0 = \rho g \Delta h = -2.66 \times 10^3$ Pa

$$P = P_0 + \Delta P_0 = (1.013 - 0.0266) \times 10^5 \text{ Pa} = \boxed{0.986 \times 10^5 \text{ Pa}}$$

15.17 $P_0 = \rho g h$

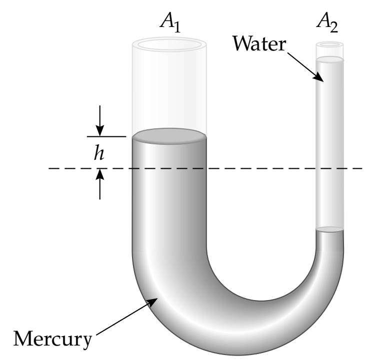
$$h = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5 \text{ Pa}}{(0.984 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{10.5 \text{ m}}$$

Some alcohol and water will evaporate.

15.18 (a) Using the definition of density, we have

$$h_w = \frac{m_{\text{water}}}{A_2 \rho_{\text{water}}} = \frac{100 \text{ g}}{(5.00 \text{ cm}^2)(1.00 \text{ g/cm}^3)} = \boxed{20.0 \text{ cm}}$$

(b) The sketch at the right represents the situation after the water is added. A volume ($A_2 h_2$) of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is $A_1 h$. Since the total volume of mercury has not changed,



(b)

$$A_2 h_2 = A_1 h$$

or $$h_2 = \frac{A_1}{A_2} h \quad (1)$$

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as:

$$P = P_0 + \rho_{\text{water}} g h_w$$

The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{\text{Hg}} g (h + h_2) = P_0 + \rho_{\text{water}} g h_w$$

which, using equation (1) above, reduces to

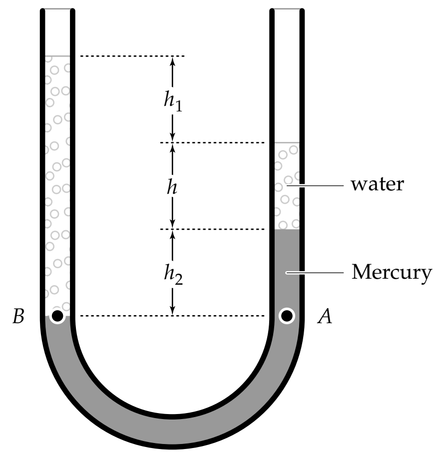
$$\rho_{\text{Hg}} h \left[1 + \frac{A_1}{A_2}\right] = \rho_{\text{water}} h_w$$

or $$h = \frac{\rho_{\text{water}} h_w}{\rho_{\text{Hg}} \left(1 + \frac{A_1}{A_2}\right)}$$

Thus, the level of mercury has risen a distance of

$$h = \frac{(1.00\ \text{g/cm}^3)(20.0\ \text{cm})}{(13.6\ \text{g/cm}^3)\left(1 + \dfrac{10.0}{5.00}\right)} = \boxed{0.490\ \text{cm}}\quad \text{above the original level.}$$

15.19 Let h be the height of the water column added to the right side of the U–tube. Then when equilibrium is reached, the situation is as shown in the sketch at right. Now consider two points, A and B shown in the sketch, at the level of the water–mercury interface. By Pascal's Principle, the absolute pressure at B is the same as that at A. But,



$$P_A = P_0 + \rho_w g h + \rho_{Hg} g h_2 \quad \text{and}$$

$$P_B = P_0 + \rho_w g(h_1 + h + h_2)$$

Thus, from $P_A = P_B$, $\rho_w h_1 + \rho_w h + \rho_w h_2 = \rho_w h + \rho_{Hg} h_2$, or

$$h_1 = \left[\frac{\rho_{Hg}}{\rho_w} - 1\right] h_2 = (13.6 - 1)(1.00\ \text{cm}) = \boxed{12.6\ \text{cm}}$$

***15.20** (a) The balloon is nearly in equilibrium:

$$\sum F_y = ma_y \Rightarrow B - (F_g)_{\text{helium}} - (F_g)_{\text{payload}} = 0$$

or $\rho_{\text{air}} g V - \rho_{\text{helium}} g V - m_{\text{payload}} g = 0$

This reduces to

$$m_{\text{payload}} = (\rho_{\text{air}} - \rho_{\text{helium}})V$$

$$= (1.29\ \text{kg/m}^3 - 0.179\ \text{kg/m}^3)(400\ \text{m}^3)$$

$$m_{\text{payload}} = \boxed{444\ \text{kg}}$$

(b) Similarly,

$$m_{\text{payload}} = (\rho_{\text{air}} - \rho_{\text{hydrogen}})V$$

$$= (1.29\ \text{kg/m}^3 - 0.0899\ \text{kg/m}^3)(400\ \text{m}^3)$$

$$m_{\text{payload}} = \boxed{480\ \text{kg}}$$

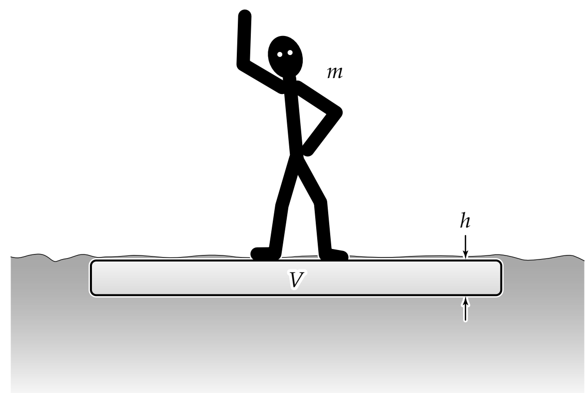
The air does the lifting, nearly the same for the two balloons.

15.21 The total weight supported is:

$$F_g = (m + \rho_s V)g$$

$$F_g = \left[75.0\text{ kg} + \left(300\ \frac{\text{kg}}{\text{m}^3}\right)(0.100\text{ m})A\right]\left(9.80\ \frac{\text{m}}{\text{s}^2}\right)$$

$$F_g = 735\text{ N} + (294\text{ N/m}^2)A$$



The buoyancy force is:

$$F_b = \rho_w g V = \left(10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.80\ \frac{\text{m}}{\text{s}^2}\right)[(0.100\text{ m})A]$$

$$F_b = (980\text{ N/m}^2)A$$

Since $F_b = F_g$,

$$(980\text{ N/m}^2)A = 735\text{ N} + (294\text{ N/m}^2)A$$

or $A = \dfrac{735\text{ N}}{(980 - 294)\text{ N/m}^2} = \boxed{1.07\text{ m}^2}$

15.22 $F_g = (m + \rho_s V)g = F_b = \rho_w V g$ (see the figure with 15.21)

Since $V = Ah$, $m + \rho_s Ah = \rho_w Ah$,

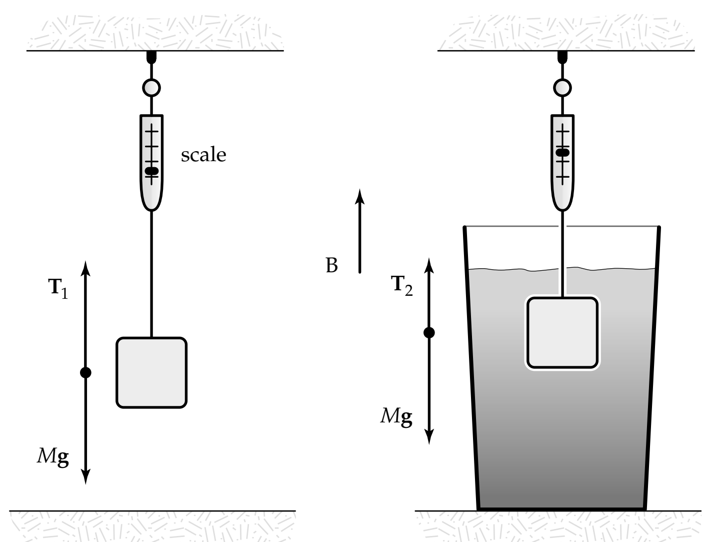
and $A = \boxed{\dfrac{m}{(\rho_w - \rho_s)h}}$

15.23 (a) Before the metal is immersed:

$\sum F_y = T_1 - Mg = 0$ or

$T_1 = Mg = (1.00\text{ kg})\left(9.80\ \dfrac{\text{m}}{\text{s}^2}\right)$

$= \boxed{9.80\text{ N}}$



(b) After the metal is immersed:

$\Sigma F_y = T_2 + B - Mg = 0$ or

$T_2 = Mg - B = Mg - (\rho_w V)g$

$V = \dfrac{M}{\rho} = \dfrac{1.00 \text{ kg}}{2700 \text{ kg/m}^3}$

Thus,

$$T_2 = Mg - B = 9.80 \text{ N} - (1000 \text{ kg/m}^3)\left(\frac{1.00 \text{ kg}}{2700 \text{ kg/m}^3}\right)\left(9.80 \frac{\text{m}}{\text{s}^2}\right) = \boxed{6.17 \text{ N}}$$

15.24 (a) $P = P_0 + \rho g h$

Taking $P_0 = 1.0130 \times 10^3 \text{ N/m}^2$ and $h = 5.00 \text{ cm}$

we find $P_{top} = 1.0179 \times 10^5 \text{ N/m}^2$

For $h = 17.0$ cm, we get $P_{bot} = 1.0297 \times 10^5 \text{ N/m}^2$

Since the areas of the top and bottom are

$A = (0.100 \text{ m} \times 0.100 \text{ m}) = 10^{-2} \text{ m}^2$

we find $F_{top} = P_{top} A = \boxed{1.0179 \times 10^3 \text{ N}}$

and $F_{bot} = \boxed{1.0297 \times 10^3 \text{ N}}$

(b) $T + B - Mg = 0$ where

$B = \rho_w V g = (10^3 \text{ kg/m}^3)(1.20 \times 10^{-3} \text{ m}^3)(9.80 \text{ m/s}^2) = 11.8 \text{ N}$

and $Mg = 10.0(9.80) = 98.0 \text{ N}$

Therefore, $T = Mg - B = 98.0 - 11.8 = \boxed{86.2 \text{ N}}$

(c) $F_{bot} - F_{top} = (1.0297 - 1.0179) \times 10^3 \text{ N} = \boxed{11.8 \text{ N}}$

which is equal to B found in part (b).

15.25 (a) According to Archimedes,

$$B = \rho_{\text{water}} Vg = (1.00 \text{ g/cm}^3)[20.0 \times 20.0 \times (20.0 - h)]g$$

But

$$B = \text{Weight of Block} = Mg = \rho_{\text{wood}} V_{\text{wood}}\, g = (0.650 \text{ g/cm}^3)(20.0 \text{ cm})^3 g$$

$$(0.650)(20.0)^3 g = (1.00)(20.0)(20.0)(20.0 - h)g$$

$$20.0 - h = 20.0(0.650)$$

$$h = 20.0(1 - 0.650) = \boxed{7.00 \text{ cm}}$$

(b) $B = F_g + Mg$ where M = mass of lead

$$(1.00)(20.0)^3 g = (0.650)(20.0)^3 g + Mg$$

$$M = (20.0)^3(1.00 - 0.650) = (20.0)^3(0.350) = 2800 \text{ g} = \boxed{2.80 \text{ kg}}$$

***15.26** Consider spherical balloons of radius 12.5 cm containing helium at STP and immersed in air at 0°C and 1 atm. If the rubber envelope has mass 5.00 g, the upward force on each is

$$B - F_{g,\text{He}} - F_{g,\text{env}} = \rho_{\text{air}} Vg - \rho_{\text{He}} Vg - m_{\text{env}}\, g$$

$$F_{\text{up}} = (\rho_{\text{air}} - \rho_{\text{He}}) \frac{4}{3} \pi r^3 g - m_{\text{env}}\, g$$

$$F_{\text{up}} = (1.29 - 0.179) \frac{\text{kg}}{\text{m}^3} \frac{4}{3} \pi (0.125 \text{ m})^3 (9.80 \text{ m/s}^2) - 5.00 \times 10^{-3} \text{ kg}(9.80 \text{ m/s}^2)$$

$$F_{\text{up}} = 0.0401 \text{ N}$$

If your weight (including harness, strings, and submarine sandwich) is

$$(70.0 \text{ kg})(9.80 \text{ m/s}^2) = 686 \text{ N}$$

you need this many balloons:

$$\frac{686 \text{ N}}{0.0401 \text{ N}} = 17\,000 \sim \boxed{10^4}$$

15.27 $\rho_{H_2O} g \frac{V}{2} = \rho_{\text{sphere}} g V$

$\rho_{\text{sphere}} = \frac{1}{2} \rho_{H_2O} = \boxed{500 \text{ kg/m}^3}$

$\rho_{\text{oil}} g \frac{4}{10} V - \rho_{\text{sphere}} g V = 0$

$\rho_{\text{oil}} = \frac{10}{4}(500 \text{ kg/m}^3) = \boxed{1250 \text{ kg/m}^3}$

15.28 Since the frog floats, the buoyant force = the weight of the frog. Also, the weight of the displaced water = weight of the frog, so

$$\rho_{\text{ooze}} V g = m_{\text{frog}} g$$

or $$m_{\text{frog}} = \rho_{\text{ooze}} V = \rho_{\text{ooze}} \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = (1.35 \times 10^3 \text{ kg/m}^3)\frac{2\pi}{3}(6.00 \times 10^{-2} \text{ m})^3$$

Hence, $m_{\text{frog}} = \boxed{0.611 \text{ kg}}$

15.29 The balloon stops rising when $(\rho_{\text{air}} - \rho_{\text{He}}) g V = M g$

or, when $(\rho_{\text{air}} - \rho_{\text{He}}) V = M$

Therefore, $V = \frac{M}{(\rho_{\text{air}} - \rho_{\text{He}})} = \frac{400}{(1.25e^{-1} - 0.180)} = \boxed{1430 \text{ m}^3}$

15.30 Let ℓ represent the length below water at equilibrium and M the tube's mass:

$$\Sigma F_y = 0 \Rightarrow -Mg + \rho \pi r^2 \ell g = 0$$

Now with any excursion x from equilibrium

$$-Mg + \rho \pi r^2 (\ell - x) g = Ma$$

Subtracting the equilibrium equation gives

$$-\rho \pi r^2 g x = Ma$$

$$a = -(\rho \pi r^2 g / M) x = -\omega^2 x$$

The opposite direction and direct proportionality of a to x imply SHM with angular frequency

$$\omega = \sqrt{\rho \pi r^2 g / M}$$

$$T = \frac{2\pi}{\omega} = \boxed{\left(\frac{2}{r}\right)\sqrt{\frac{\pi M}{\rho g}}}$$

***15.31** Constant velocity implies zero acceleration, which means that the submersible is in equilibrium under the gravitational force, the upward buoyant force, and the upward resistance force:

$$\Sigma F_y = ma_y = 0$$

$$-(1.20 \times 10^4 \text{ kg} + m)g + \rho_w g V + 1100 \text{ N} = 0$$

where m is the mass of the added water and V is the sphere's volume.

$$(1.20 \times 10^4 \text{ kg} + m) = (1.03 \times 10^3)\left(\frac{4\pi}{3}\right)(1.50)^3 + \left(\frac{1100 \text{ N}}{9.80 \text{ m/s}^2}\right)$$

so $\boxed{m = 2.67 \times 10^3 \text{ kg}}$

***15.32** By Archimedes's principle, the weight of the fifty planes is equal to the weight of a horizontal slice of water 11.0 cm thick and circumscribed by the water line:

$$\Delta B = \rho_{\text{water}} g(\Delta V)$$

$$50(2.90 \times 10^4 \text{ kg})g = (1030 \text{ kg/m}^3)g(0.110 \text{ m})A$$

giving $A = \boxed{1.28 \times 10^4 \text{ m}^2}$ The acceleration of gravity does not affect the answer.

***15.33** Volume flow rate $= A_1 v_1 = A_2 v_2$

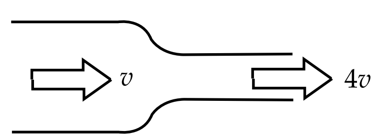
$$\frac{20.0 \text{ L}}{60.0 \text{ s}}\left(\frac{1000 \text{ cm}^3}{1.00 \text{ L}}\right) = \pi(1.00 \text{ cm})^2 v_{\text{hose}} = \pi(0.500 \text{ cm})^2 v_{\text{nozzle}}$$

(a) $v_{\text{hose}} = \dfrac{333 \text{ cm}^3\text{/s}}{3.14 \text{ cm}^2} = \boxed{106 \text{ cm/s}}$

(b) $v_{\text{nozzle}} = \dfrac{333 \text{ cm}^3\text{/s}}{0.785 \text{ cm}^2} = \boxed{424 \text{ cm/s}}$

15.34 By Bernoulli's equation,

$$8.00 \times 10^4 \frac{\text{N}}{\text{m}^2} + \frac{1}{2} 1000v^2 = 6.00 \times 10^4 \frac{\text{N}}{\text{m}^2} + \frac{1}{2} 1000(16v^2)$$



$$2.00 \times 10^4 \frac{\text{N}}{\text{m}^2} = \frac{1}{2} 1000(15v^2)$$

$$v = 1.63 \text{ m/s}$$

$$\frac{dm}{dt} = \rho A v = 1000\pi(5.00 \times 10^{-2})^2(1.63 \text{ m/s}) = \boxed{12.8 \text{ kg/s}}$$

15.35 Assuming the top is open to the atmosphere, then

$P_1 = P_0$

Note $P_2 = P_0$

Flow rate $= 2.50 \times 10^{-3}\ \text{m}^3/\text{min} = 4.17 \times 10^{-5}\ \text{m}^3/\text{s}$

(a) $A_1 >> A_2$ so $v_1 << v_2$

Assuming $v_1 \approx 0$,

$$P_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = P_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$

$$v_2 = (2gy_1)^{1/2} = [2(9.80)(16.0)]^{1/2} = \boxed{17.7\ \text{m/s}}$$

(b) Flow rate $= A_2 v_2 = \left(\frac{\pi d^2}{4}\right)(17.7) = 4.17 \times 10^{-5}\ \text{m}^3/\text{s}$

$$d = \boxed{1.73 \times 10^{-3}\ \text{m}} = 1.73\ \text{mm}$$

15.36 (a) Suppose the flow is very slow

$$\left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{river}} = \left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{rim}}$$

$$P + 0 + \rho g(564\ \text{m}) = 1\ \text{atm} + 0 + \rho g(2096\ \text{m})$$

$$P = 1\ \text{atm} + 1000\ \text{kg/m}^3(9.80\ \text{m/s}^2)1532\ \text{m}$$

$$P = \boxed{1\ \text{atm} + 15.0\ \text{MPa}}$$

(b) The volume flow rate is

$$\frac{4500\ \text{m}^3}{\text{d}} = Av = \frac{\pi d^2 v}{4}$$

$$v = \frac{4500\ \text{m}^3}{\text{d}}\left(\frac{1\ \text{d}}{86400\ \text{s}}\right)\frac{4}{\pi(0.150\ \text{m})^2}$$

$$v = \boxed{2.98\ \text{m/s}}$$

(c) Imagine the pressure as applied to stationary water at the bottom of the pipe:

$$\left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{bottom}} = \left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{top}}$$

$$P + 0 = 1 \text{ atm} + \frac{1}{2}(1000 \text{ kg/m}^3)(2.98 \text{ m/s})^2 + (1000 \text{ kg})9.80 \text{ m/s}^2(1532 \text{ m})$$

$$P = 1 \text{ atm} + 15.0 \text{ MPa} + 4.45 \text{ kPa}$$

The additional pressure is $\boxed{4.45 \text{ kPa}}$

15.37 Flow rate $Q = 0.0120 \text{ m}^3/\text{s} = v_1 A_1 = v_2 A_2$

$$v_2 = \frac{Q}{A_2} = \frac{0.0120}{A_2} = \boxed{31.6 \text{ m/s}}$$

***15.38** (a) For upward flight of a water–drop projectile from geyser vent to fountain–top:

$$v_{yf}^2 = v_{yi}^2 + 2a_y(\Delta y)$$

$$0 = v_i^2 + 2(-9.80 \text{ m/s}^2)(+40.0 \text{ m})$$

$$v_i = \boxed{28.0 \text{ m/s}}$$

(b) Between geyser vent and fountain–top:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Air is so low in density that very nearly $P_1 = P_2 = 1$ atm. Then,

$$\frac{1}{2}v_1^2 + 0 = 0 + (9.80 \text{ m/s}^2)(40.0 \text{ m}) \qquad v_1 = \boxed{28.0 \text{ m/s}}$$

(c) Between the chamber and the fountain–top:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_1 + 0 + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(-175 \text{ m})$$

$$= P_0 + 0 + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(+40.0 \text{ m})$$

$$P_1 - P_0 = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(215 \text{ m}) = \boxed{2.11 \text{ MPa}}$$

15.39 $Mg = (P_1 - P_2)A$ for a balanced condition

$$\frac{(16000)(9.80)}{A} = 7.00 \times 10^4 - P_2$$

where $A = 80.0\ \text{m}^2$,

$$\therefore\ P_2 = 7.00 \times 10^4 - 0.196 \times 10^4 = \boxed{6.80 \times 10^4\ \text{Pa}}$$

15.40 $P_1 + \dfrac{\rho v_1^2}{2} = P_2 + \dfrac{\rho v_2^2}{2}$ (Bernoulli equation)

$v_1 A_1 = v_2 A_2$ where $\dfrac{A_1}{A_2} = 4$

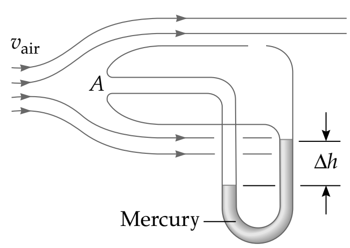
$$\Delta P = P_1 - P_2 = \frac{\rho}{2}(v_2^2 - v_1^2) = \frac{\rho}{2} v_1^2 \left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$\Delta P = \frac{\rho v_1^2}{2}(15) = 21\,000\ \text{Pa}$$

$v_1 = 2.00\ \text{m/s}$

$v_2 = 4v_1 = 8.00\ \text{m/s}$

and $Q = v_1 A_1 = \boxed{2.51 \times 10^{-3}\ \text{m}^3/\text{s}}$



15.41 $\rho_{\text{Air}} \dfrac{v^2}{2} = \Delta P = \rho_{\text{Hg}}\, g\, \Delta h$

$$v = \sqrt{\frac{2\rho_{\text{Hg}}\, g \Delta h}{\rho_{\text{Air}}}} = \boxed{103\ \text{m/s}}$$

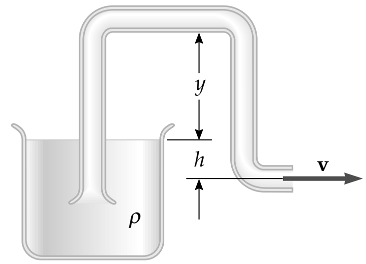
***15.42** The assumption of incompressibility is surely unrealistic, but allows an estimate of the speed:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$1.00\ \text{atm} + 0 + 0 = 0.287\ \text{atm} + 0 + \frac{1}{2}(1.20\ \text{kg/m}^3)\, v_2^2$$

$$v_2 = \sqrt{\frac{2(1.00 - 0.287)(1.03 \times 10^5\ \text{N/m}^2)}{1.20\ \text{kg/m}^3}} = \boxed{347\ \text{m/s}}$$

15.43 (a) $P_0 + \rho g h + 0 = P_0 + 0 + \frac{1}{2}\rho v_3^2$



$v_3 = \sqrt{2gh}$

If $h = 1.00$ m,

$v_3 = \boxed{4.43 \text{ m/s}}$

(b) $P + \rho g y + \frac{1}{2}\rho v_2^2 = P_0 + 0 + \frac{1}{2}\rho v_3^2$

Since $v_2 = v_3$,

$P = P_0 - \rho g y$

Since $P \geq 0$,

$$y \leq \frac{P_0}{\rho g} = \frac{(1.013 \times 10^5 \text{ Pa})}{(10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{10.3 \text{ m}}$$

***15.44** In the reservoir, the gauge pressure is

$$\Delta P = \frac{2.00 \text{ N}}{2.50 \times 10^{-5} \text{ m}^2} = 8.00 \times 10^4 \text{ Pa}$$

From the equation of continuity:

$A_1 v_1 = A_2 v_2$

$(2.50 \times 10^{-5} \text{ m}^2)v_1 = (1.00 \times 10^{-8} \text{ m}^2)v_2$

$v_1 = (4.00 \times 10^{-4})v_2$

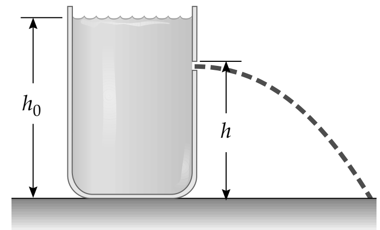
Thus, v_1^2 is negligible in comparison to v_2^2. Then, from Bernoulli's equation:

$(P_1 - P_2) + \rho g y_1 + \frac{1}{2}\rho v_1^2 = \rho g y_2 + \frac{1}{2}\rho v_2^2$

$8.00 \times 10^4 \text{ Pa} + 0 + 0 = 0 + \frac{1}{2}(1000 \text{ kg/m}^3)\, v_2^2$

$$v_2 = \sqrt{\frac{2(8.00 \times 10^4 \text{ Pa})}{1000 \text{ kg/m}^3}} = \boxed{12.6 \text{ m/s}}$$

15.45 Apply Bernoulli's equation between the top surface and the exiting stream.



$$P_0 + 0 + \rho g h_0 = P_0 + \frac{1}{2}\rho v_x^2 + \rho g h$$

$$v_x^2 = 2g(h_0 - h) \qquad \therefore v_x = \sqrt{2g(h_0 - h)}$$

$$x = v_x t$$

$$y = h = \frac{1}{2}gt^2$$

$$\therefore t = \sqrt{\frac{2h}{g}}$$

and $x = v_x\sqrt{\frac{2h}{g}} = \sqrt{2g(h_0 - h)}\sqrt{\frac{2h}{g}}$

$$x = \boxed{2\sqrt{h(h_0 - h)}}$$

15.46 $x = v_x t \qquad h = \frac{1}{2}gt^2$

$v_x = \sqrt{2g(h_0 - h)}$ from Problem 45

and $t = \sqrt{\frac{2h}{g}}$

$$x = \sqrt{2g(h_0 - h)}\sqrt{\frac{2h}{g}} = \sqrt{4h(h_0 - h)}$$

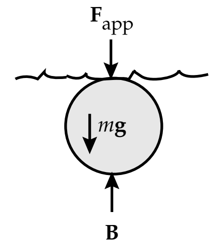
(a) Maximize x with respect to h:

$$\frac{dx}{dh} = 0$$

$$\frac{dx}{dh} = \frac{\frac{1}{2}(4h_0 - 8h)}{\sqrt{4h(h_0 - h)}} = 0 \quad \text{when } \boxed{h = \frac{h_0}{2}}$$

(b) For $h = \frac{h_0}{2}$, $\quad v_x = \sqrt{gh_0}$, $\quad t = \sqrt{\frac{h_0}{g}}$, $\quad$ then $\quad x = v_x t = \boxed{h_0}$

15.47 At equilibrium $\sum F = 0$



or $\mathbf{F}_{app} + m\mathbf{g} = \mathbf{B}$ where $\mathbf{B}$ is the buoyant force

The applied force,

$$\mathbf{F}_{app} = \mathbf{B} - m\mathbf{g} \quad \text{where } \mathbf{B} \equiv (\text{Vol})(\rho_{water})g$$

and $m = (\text{Vol})\rho_{ball}$

So, $F_{app} = (\text{Vol})g(\rho_{water} - \rho_{ball}) = \frac{4}{3}\pi r^3 g(\rho_{water} - \rho_{ball})$

$$F_{app} = \frac{4}{3}\pi(1.90 \times 10^{-2}\text{ m})^3(9.80\text{ m/s}^2)(10^3\text{ kg/m}^3 - 84.0\text{ kg/m}^3)$$

$$\boxed{F_{app} = 0.258\text{ N}}$$

<u>Goal Solution</u>

G: According to Archimedes's Principle, the buoyant force acting on the submerged the ball will be equal to the weight of the water the ball will displace. The ball has a volume of about 30 cm^3, so the weight of this water is approximately:

$$B = F_g = \rho V g \approx (1\text{ g/cm}^3)(30\text{ cm}^3)(10\text{ m/s}^2) = 0.3\text{ N}$$

Since the ball is much less dense than the water, the applied force will approximately equal this buoyant force.

O: Apply Newton's 2nd law to find the applied force.

A: At equilibrium, $\sum F = 0$ or $F_{app} + mg - B = 0$

Where the buoyant force is $B = \rho_w V g$ and $\rho_w = 1000\text{ kg/m}^3$

The applied force is then, $F_{app} = \rho_w V g - mg$

Using $m = \rho_{ball} V$ to eliminate the unknown mass of the ball, this becomes

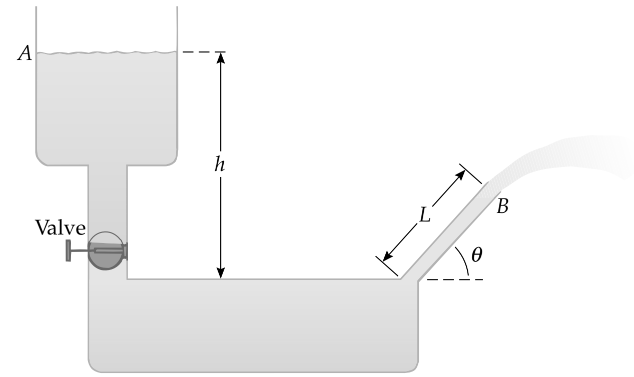
$$F_{app} = Vg(\rho_w - \rho_{ball}) = \frac{4}{3}\pi r^3 g(\rho_w - \rho_{ball})$$

$$F_{app} = \frac{4}{3}\pi(1.90 \times 10^{-2}\text{ m})^3(9.80\text{ m/s}^2)(10^3\text{ kg/m}^3 - 84\text{ kg/m}^3)$$

$$F_{app} = 0.258\text{ N}$$

L: The force is approximately what we expected, so our result is reasonable. If the applied force would be greater than 0.258 N, the ball would sink until it hit the bottom (which would then provide a normal force directed upwards).

15.48 Consider the diagram and apply Bernoulli's equation to points A and B, taking $y = 0$ at the level of point B, and recognizing that v_A is approximately zero.



This gives:

$$P_A + \frac{1}{2}\rho_w(0)^2 + \rho_w g(h - L\sin\theta)$$

$$= P_B + \frac{1}{2}\rho_w v_B^2 + \rho_w g(0)$$

Now, recognize that $P_A = P_B = P_{\text{atmosphere}}$ since both points are open to the atmosphere (neglecting variation of atmospheric pressure with altitude). Thus, we obtain

$$v_B = \sqrt{2g(h - L\sin\theta)} = \sqrt{2(9.80\text{ m/s}^2)[10.0\text{ m} - (2.00\text{ m})\sin 30.0°]}$$

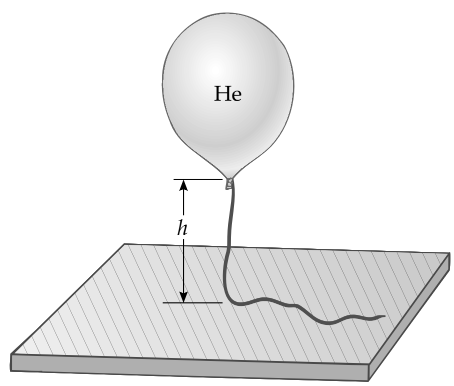
$$v_B = 13.3\text{ m/s}$$

Now the problem reduces to one of projectile motion with $v_{yi} = v_B \sin 30.0° = 6.64$ m/s.. Then, $v_y^2 = v_{yi}^2 + 2a(\Delta y)$ gives at the top of the arc (where $y = y_{max}$ and $v_y = 0$)

$$0 = (6.64\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(y_{max} - 0)$$

or $y_{max} =$
Error!

15.49 When the balloon comes into equilibrium, we must have



$$\Sigma F_y = B - F_{g,\text{ balloon}} - F_{g,\text{He}} - F_{g,\text{string}} = 0$$

$F_{g,\text{ string}}$ is the weight of the string above the ground, and B is the buoyant force. Now

$$F_{g,\text{ balloon}} = m_{\text{balloon}}\, g$$

$$F_{g,\text{ He}} = \rho_{\text{He}} V g$$

$$B = \rho_{\text{air}} V g$$

and $F_{g,\text{ string}} = m_{\text{string}} \frac{h}{L} g$

Therefore, we have

$$\rho_{\text{air}}\, Vg - m_{\text{balloon}}\, g - \rho_{\text{He}}\, Vg - m_{\text{string}}\frac{h}{L}\, g = 0$$

or $$h = \frac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{m_{\text{string}}}\, L$$

giving,

$$h = \frac{(1.29 - 0.179)\ \text{kg/m}^3\left(\frac{4\pi(0.400\ \text{m})^3}{3}\right) - 0.250\ \text{kg}}{0.0500\ \text{kg}}(2.00\ \text{m}) = \boxed{1.91\ \text{m}}$$

15.50 Assume $v_{\text{inside}} \approx 0$

$$P + 0 + 0 = 1\ \text{atm} + \frac{1}{2}(1000)(30.0\ \text{m/s})^2 + 1000(9.80)(0.500)$$

$$P_{\text{gauge}} = P - 1\ \text{atm} = 4.50 \times 10^5 + 4.90 \times 10^3 = \boxed{455\ \text{kPa}}$$

15.51 The "balanced" condition is one in which the apparent weight of the body equals the apparent weight of the weights. This condition can be written as:

$$F_g - B = F_g' - B'$$

where B and B' are the buoyant forces on the body and weights respectively. The buoyant force experienced by an object of volume V in air equals:

Buoyant force = (Volume of object)$\rho_{\text{air}}g$

so we have $B = V\rho_{\text{air}}g$ and $B' = \left(\frac{F_g'}{\rho g}\right)\rho_{\text{air}}g$

Therefore, $F_g = \boxed{F_g' + \left(V - \frac{F_g'}{\rho g}\right)\rho_{\text{air}}g}$

<u>Goal Solution</u>

The "balanced" condition is one in which the net torque on the balance is zero. Since the balance has lever arms of equal length, the total force on each pan is equal. Applying $\sum\tau = 0$ around the pivot leads to

$$F_g - B = F_g' - B'$$

where B and B' are the buoyant forces on the body and weights respectively. The buoyant force experienced by an object of volume V in air is

$$B = V\rho_{\text{air}}g \quad \text{and} \quad B' = V'\rho_{\text{air}}g$$

Since the volume of the weights is not given explicitly, we must use the density equation to eliminate it:

$$V' = \frac{m'}{\rho} = \frac{m'g}{\rho g} = \frac{F_g'}{\rho g}$$

With this substitution, the buoyant force on the weights becomes

$$B' = \left(\frac{F_g'}{\rho g}\right)\rho_{\text{air}} g$$

Therefore, $F_g = F_g' + \left(V - \frac{F_g'}{\rho g}\right)\rho_{\text{air}} g$

Side note: We can now answer the popular riddle: Which weighs more, a pound of feathers or a pound of bricks?

Answer: Like in the problem above, the feathers have a greater buoyant force than the bricks , so if they "weigh" the same on a scale as a pound of bricks, then the feathers must have more mass and therefore a greater "true weight."

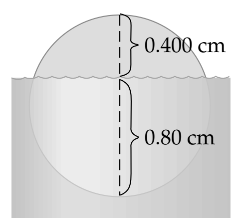
15.52 $P = \rho g h$

$1.013 \times 10^5 = (1.29)(9.80)h$

$h = \boxed{8.01 \text{ km}}$

For Mt. Everest, 29 300 ft = 8.88 km $\boxed{\text{Yes}}$

15.53 The cross–sectional area above water is



$$\frac{2.46 \text{ rad}}{2\pi}\pi(0.600 \text{ cm})^2 - (0.200 \text{ cm})(0.566 \text{ cm}) = 0.330 \text{ cm}^2$$

$A_{\text{all}} = \pi(0.600)^2 = 1.13 \text{ cm}^2$

$\rho_{\text{water}} g A_{\text{under}} = \rho_{\text{wood}} A_{\text{all}} g$

$$\rho_{\text{wood}} = \frac{1.13 - 0.330}{1.13} = 0.709 \text{ g/cm}^3 = \boxed{709 \text{ kg/m}^3}$$

15.54 At equilibrium, $\sum F_y = 0$

or $B - F_{spring} - F_{g,\,He} - F_{g,\,balloon} = 0$

giving $F_{spring} = kL = B - (m_{He} + m_{balloon})g$

But B = weight of displaced air = $\rho_{air} Vg$ and $m_{He} = \rho_{He} V$

Therefore, we have:

$$kL = \rho_{air} Vg - \rho_{He} Vg - m_{balloon} g$$

or $$L = \frac{(\rho_{air} - \rho_{He})V - m_{balloon}}{k} g$$

From the given data, this gives

$$L = \frac{(1.29\ \text{kg/m}^3 - 0.180\ \text{kg/m}^3)(5.00\ \text{m}^3) - 2.00\times 10^{-3}\ \text{kg}}{90.0\ \text{N/m}}(9.80) = \boxed{0.604\ \text{m}}$$

15.55 Looking first at the top scale and the iron block, we have:

$$T_1 + B = F_{g,\,iron}$$

where T_1 is the tension in the spring scale, B is the buoyant force, and $F_{g,\,iron}$ is the weight of the iron block. Now if m_{iron} is the mass of the iron block, we have

$$m_{iron} = \rho_{iron} V \quad \text{so} \quad V = \frac{m_{iron}}{\rho_{iron}} = V_{displaced\ oil}$$

Then, $B = \rho_{oil} V_{iron}\, g$

Therefore, $T_1 = F_{g,\,iron} - \rho_{oil} V_{iron}\, g = m_{iron}\, g - \rho_{oil} \dfrac{m_{iron}}{\rho_{iron}} g$

or $$T_1 = \left(1 - \frac{\rho_{oil}}{\rho_{iron}}\right) m_{iron}\, g = \left(1 - \frac{916}{7860}\right)(2.00)(9.80) = \boxed{17.3\ \text{N}}$$

Next, we look at the bottom scale which reads T_2 (i.e., exerts an upward force T_2 on the system). Consider the external vertical forces acting on the beaker–oil–iron combination.

$\sum F_y = 0$ gives

$$T_1 + T_2 - F_{g,\,beaker} - F_{g,\,oil} - F_{g,\,iron} = 0$$

or $$T_2 = (m_{beaker} + m_{oil} + m_{iron})g - T_1 = (5.00\ \text{kg})(9.80\ \text{m/s}^2) - 17.3\ \text{N}$$

Thus, $T_2 = \boxed{31.7\ \text{N}}$ is the lower scale reading.

15.56 Looking at the top scale and the iron block:

$$T_1 + B = F_{g,\text{Fe}} \quad \text{where} \quad B = \rho_0 V_{\text{Fe}} g = \rho_0 \left(\frac{m_{\text{Fe}}}{\rho_{\text{Fe}}}\right) g$$

is the buoyant force exerted on the iron block by the oil. Thus,

$$T_1 = F_{g,\text{Fe}} - B = m_{\text{Fe}} g - \rho_0 \left(\frac{m_{\text{Fe}}}{\rho_{\text{Fe}}}\right) g$$

or $\boxed{T_1 = \left(1 - \dfrac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}} g}$ is the reading on the top scale.

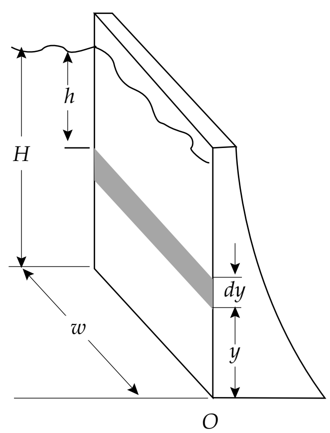
Now, consider the bottom scale which exerts an upward force of T_2 on the beaker–oil–iron combination.

$$\Sigma F_y = 0 \Rightarrow T_1 + T_2 - F_{g,\text{beaker}} - F_{g,\text{oil}} - F_{g,\text{Fe}} = 0$$

$$T_2 = F_{g,\text{beaker}} + F_{g,\text{oil}} + F_{g,\text{Fe}} - T_1 = (m_b + m_0 + m_{\text{Fe}})g - \left(1 - \frac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}} g$$

or $\boxed{T_2 = \left[m_b + m_0 + \left(\dfrac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}}\right] g}$ is the reading on the bottom scale.

15.57 The torque is



$$\tau = \int d\tau = \int r dF$$

From the figure,

$$\tau = \int_0^H y[\rho g(H - y) w dy] = \boxed{\frac{1}{6} \rho g w H^3}$$

The total force is given as

$$\frac{1}{2} \rho g w H^2$$

If this were applied at a height y_{eff} such that the torque remains unchanged, we have

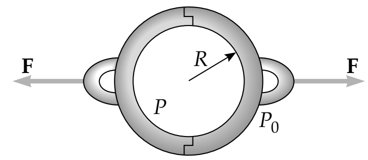
$$\frac{1}{6} \rho g w H^3 = y_{\text{eff}} \left[\frac{1}{2} \rho g w H^2\right]$$

and $\boxed{y_{\text{eff}} = \dfrac{1}{3} H}$

15.58 (a) The pressure on the surface of the two hemispheres is constant at all points, and the force on each element of surface area is directed along the radius of the hemispheres. The applied force along the axis must balance the force on the "effective" area which is the projection of the actual surface onto a plane perpendicular to the x axis,

$$A = \pi R^2$$

Therefore,



$$F = \boxed{(P_0 - P)\pi R^2}$$

(b) For the values given

$$F = (P_0 - 0.100P_0)\pi(0.300\text{ m})^2 = 0.254P_0 = \boxed{2.58 \times 10^4\text{ N}}$$

15.59 $\rho_{Cu} V = 3.083\text{ g}$

$\rho_{Zn}(xV) + \rho_{Cu}(1 - x)V = 2.517\text{ g}$

$$\rho_{Zn}\left(\frac{3.083}{\rho_{Cu}}\right) x + 3.083(1 - x) = 2.517$$

$$\left(1 - \frac{7.133}{8.960}\right) x = \left(1 - \frac{2.517}{3.083}\right)$$

$x = 0.9004$

$\%\text{Zn} = \boxed{90.04\%}$

15.60 (a) From

$$\sum F = ma$$

$$B - m_{shell}\,g - m_{He}\,g = m_{total}\,a = (m_{shell} + m_{He})a \quad (1)$$

Where $B = \rho_{water} Vg$ and $m_{He} = \rho_{He} V$

Also, $V = \frac{4}{3}\pi r^3 = \frac{\pi d^3}{6}$

Putting these into equation (1) above,

$$\left(m_{shell} + \rho_{He}\frac{\pi d^3}{6}\right) a = \left(\rho_{water}\frac{\pi d^3}{6} - m_{shell} - \rho_{He}\frac{\pi d^3}{6}\right) g$$

which gives

$$a = \frac{(\rho_{\text{water}} - \rho_{\text{He}})\frac{\pi d^3}{6} - m_{\text{shell}}}{m_{\text{shell}} + \rho_{\text{He}}\frac{\pi d^3}{6}}\, g$$

or $$a = \frac{(1000 - 0.180)\left(\frac{\text{kg}}{\text{m}^3}\right)\frac{\pi(0.200\text{ m})^3}{6} - 4.00\text{ kg}}{4.00\text{ kg} + \left(0.180\,\frac{\text{kg}}{\text{m}^3}\right)\frac{\pi(0.200\text{ m})^3}{6}}\, 9.80\text{ m/s}^2 = \boxed{0.461\text{ m/s}^2}$$

(b) $$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(h - d)}{a}} = \sqrt{\frac{2(4.00\text{ m} - 0.200\text{ m})}{0.461\text{ m/s}^2}} = \boxed{4.06\text{ s}}$$

15.61 Energy is conserved

$$(K + U)_i + \Delta E = (K + U)_f$$

$$0 + \frac{mgL}{2} + 0 = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{gL} = \sqrt{2.00\text{ m}(9.80\text{ m/s}^2)} = \boxed{4.43\text{ m/s}}$$

***15.62** Inertia of the disk: $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(0.250\text{ m})^2 = 0.312\text{ kg}\cdot\text{m}^2$

Angular acceleration: $\omega_f = \omega_i + \alpha t$

$$\alpha = \left(\frac{0 - 300\text{ rev/min}}{60.0\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = -0.524\text{ rad/s}^2$$

Braking torque: $\sum\tau = I\alpha \Rightarrow -fd = I\alpha$, so $f = \frac{-I\alpha}{d}$

Friction force: $f = \frac{(0.312\text{ kg}\cdot\text{m}^2)(0.524\text{ rad/s}^2)}{0.220\text{ m}} = 0.744\text{ N}$

Normal force: $f = \mu_k n \Rightarrow n = \frac{f}{\mu_k} = \frac{0.744\text{ N}}{0.500} = 1.49\text{ N}$

gauge pressure: $P = \frac{n}{A} = \frac{1.49\text{ N}}{\pi(2.50\times10^{-2}\text{ m})^2} = \boxed{758\text{ Pa}}$

***15.63** (a) We imagine the superhero to produce a perfect vacuum in the straw. Take point 1 at the water surface in the basin and point 2 at the water surface in the straw:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$1.013 \times 10^5 \text{ N/m}^2 + 0 + 0 = 0 + 0 + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2) y_2$$

$$y_2 = \boxed{10.3 \text{ m}}$$

(b) No atmosphere can lift the water in the straw through $\boxed{\text{zero}}$ height difference.

15.64 Differentiating $P = \rho g y$ we have

$$\frac{dP}{dy} = -\rho g$$

Also at any given height the density of air is proportional to pressure, or

$$\frac{P}{\rho} = \frac{P_0}{\rho_0}$$

Combining these two equations we have

$$\int_{P_0}^{P} \frac{dP}{P} = -g\frac{\rho_0}{P_0}\int_0^h dy$$

and integrating gives

$$P = P_0 e^{-\alpha h}$$

15.65 Let s stand for the edge of the cube, h for the depth of immersion, ρ_{ice} stand for the density of the ice, ρ_w stand for density of water, and ρ_a stand for density of the alcohol.

(a) According to Archimedes's principle, at equilibrium we have

$$\rho_{\text{ice}} g s^3 = \rho_w g h s^2 \Rightarrow h = s\,\frac{\rho_{\text{ice}}}{\rho_w}$$

With

$$\rho_{\text{ice}} = 0.917 \times 10^3 \text{ kg/m}^3$$

$$\rho_w = 1.00 \times 10^3 \text{ kg/m}^3$$

and $s = 20.0 \text{ mm}$

we get $h = 20.0(0.917) = 18.34 \text{ mm} \approx \boxed{18.3 \text{ mm}}$

(b) We assume that the top of the cube is still above the alcohol surface. Letting h_a stand for the thickness of the alcohol layer, we have

$$\rho_a g s^2 h_a + \rho_w g s^2 h_w = \rho_{\text{ice}} g s^3 \Rightarrow h_w = \left(\frac{\rho_{\text{ice}}}{\rho_w}\right) s - \left(\frac{\rho_a}{\rho_w}\right) h_a$$

With $\rho_a = 0.806 \times 10^3 \text{ kg/m}^3$ and $h_a = 5.00 \text{ mm}$

we obtain $h_w = 18.34 - (0.806)(5.00) = 14.31 \text{ mm} \approx \boxed{14.3 \text{ mm}}$

(c) Here $h_w' = s - h_a'$, so Archimedes's principle gives . . .

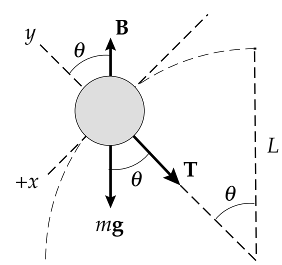
$$\rho_a g s^2 h_a' + \rho_w g s^2 (s - h_a') = \rho_{\text{ice}} g s^3 \Rightarrow \rho_a h_a' + \rho_w (s - h_a') = \rho_{\text{ice}} s$$

$$\Rightarrow h_a' = \frac{s(\rho_w - \rho_{\text{ice}})}{(\rho_w - \rho_a)} = 20.0\frac{(1.000 - 0.917)}{(1.000 - 0.806)} = 8.557 \approx \boxed{8.56 \text{ mm}}$$

15.66 (a) A study of the forces on the balloon shows that the tangential restoring force is given as:

$$F_x = -B \sin\theta + mg \sin\theta = -(B - mg)\sin\theta$$

But $B = \rho_{\text{air}} V g$ and $m = \rho_{\text{He}} V$



Also,

$$\sin\theta \approx \theta \text{ (for small } \theta)$$

so $F_x \approx -(\rho_{\text{air}} V g - \rho_{\text{He}} V g)\theta = -(\rho_{\text{air}} - \rho_{\text{He}}) V g \theta$

But $\theta = \frac{s}{L}$

and $\boxed{F_x = -(\rho_{\text{air}} - \rho_{\text{He}}) V g \frac{s}{L} = -ks}$

with $k = (\rho_{\text{air}} - \rho_{\text{He}}) \frac{Vg}{L}$

(b) Then

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\rho_{\text{He}} V}{(\rho_{\text{air}} - \rho_{\text{He}})\frac{Vg}{L}}} = 2\pi\sqrt{\frac{\rho_{\text{He}} L}{(\rho_{\text{air}} - \rho_{\text{He}}) g}}$$

giving

$$T = 2\pi\sqrt{\frac{(0.180)(3.00 \text{ m})}{(1.29 - 0.180)(9.80 \text{ m/s}^2)}} = \boxed{1.40 \text{ s}}$$

15.67 (a) The flow rate, Av, as given may be expressed as follows:

$$25.0 \text{ liters}/30.0 \text{ s} = 0.833 \text{ liters/s} = 833 \text{ cm}^3/\text{s}$$

The area of the faucet tap is π cm², so we can find the velocity as

$$v = \frac{\text{flow rate}}{A} = \frac{833 \text{ cm}^3/\text{s}}{\pi \text{ cm}^2} = 265 \text{ cm/s} = \boxed{2.65 \text{ m/s}}$$

(b) We choose point 1 to be in the entrance pipe and point 2 to be at the faucet tap. $A_1v_1 = A_2v_2$ gives $v_1 = 0.295$ m/s. Bernoulli's equation is:

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1), \text{ and gives}$$

$$P_1 - P_2 = \frac{1}{2}(10^3 \text{ kg/m}^3)\ [(2.65 \text{ m/s})^2 - (0.295 \text{ m/s})^2]$$

$$+ (10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^3)(2.00 \text{ m})$$

or $P_{\text{gauge}} = P_1 - P_2 = \boxed{2.31 \times 10^4 \text{ Pa}}$

15.68 (a) Since the upward buoyant force is balanced by the weight of the sphere,

$$m_1 g = \rho V g = \rho\left(\frac{4}{3}\pi R^3\right) g$$

In this problem, $\rho = 0.789\,45$ g/cm³ at 20.0°C, and $R = 1.00$ cm, so we find:

$$m_1 = \rho\left(\frac{4}{3}\pi R^3\right) = (0.789\,45 \text{ g/cm}^3)\left(\frac{4}{3}\pi R^3\right)(1.00 \text{ cm})^3 = \boxed{3.307 \text{ g}}$$

(b) Following the same procedure as in part (a), with $\rho' = 0.780\,97$ g/cm³ at 30.0°C, we find:

$$m_2 = \rho'\left(\frac{4}{3}\pi R^3\right) = (0.780\,97 \text{ g/cm}^3)\left(\frac{4}{3}\pi R^3\right)(1.00 \text{ cm})^3$$

or $m_2 = \boxed{3.271 \text{ g}}$

(c) When the first sphere is resting on the bottom of the tube,

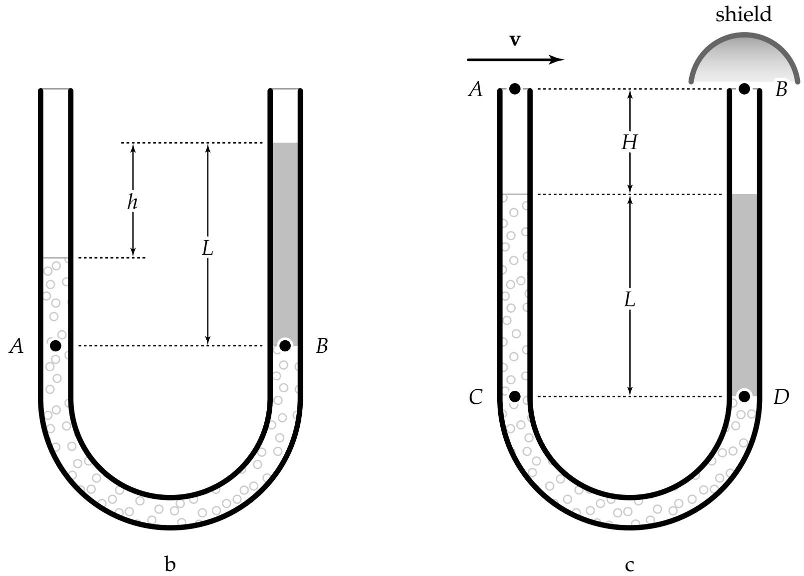
$n + B = F_{g1} = m_1 g$, where n is the normal force.

Since $B = \rho' V g$,

$$n = m_1 g - \rho' V g = \left[3.307 \text{ g} - \left(0.780\,97 \frac{\text{g}}{\text{cm}^3}\right)(1.00 \text{ cm}^3)\right]\left(980 \frac{\text{cm}}{\text{s}^2}\right)$$

$$n = 34.8 \text{ g} \cdot \text{cm/s}^2 = \boxed{3.48 \times 10^{-4} \text{ N}}$$

15.69 **Note**: Variation of atmospheric pressure with altitude is included in this solution. Because of the small distances involved, this effect is unimportant in the final answers.



(a) Consider the pressure at points A and B in part (b) of the figure:

Using the left tube: $P_A = P_{atm} + \rho_a g h + \rho_w g(L - h)$, where the second term is due to the variation of air pressure with altitude.

Using the right tube: $P_B = P_{atm} + \rho_0 g L$

But Pascal's principle says that $P_A = P_B$. Therefore,

$$P_{atm} + \rho_0 g L = P_{atm} + \rho_a g h + \rho_w g(L - h) \quad \text{or}$$

$(\rho_w - \rho_a)h = (\rho_w - \rho_0)L$, giving

$$h = \left(\frac{\rho_w - \rho_0}{\rho_w - \rho_a}\right) L = \left(\frac{1000 - 750}{1000 - 1.29}\right)(5.00 \text{ cm}) = \boxed{1.25 \text{ cm}}$$

(b) Consider part (c) of the diagram showing the situation when the air flow over the left tube equalizes the fluid levels in the two tubes. First, apply Bernoulli's equation to points A and B ($y_A = y_B$, $v_A = v$, and $v_B = 0$).

This gives: $P_A + \frac{1}{2}\rho_a v^2 + \rho_a g y_A = P_B + \frac{1}{2}\rho_a (0)^2 + \rho_a g y_B$

and since $y_A = y_B$, this reduces to: $P_B - P_A = \frac{1}{2}\rho_a v^2$ (1)

Now consider points C and D, both at the level of the oil–water interface in the right tube. Using the variation of pressure with depth in static fluids, we have:

$$P_C = P_A + \rho_a g H + \rho_w g L \quad \text{and} \quad P_D = P_B + \rho_a g H + \rho_0 g L$$

But Pascal's principle says that $P_C = P_D$. Equating these two gives:

$$P_B + \rho_a gH + \rho_0 gL = P_A + \rho_a gH + \rho_w gL \quad \text{or}$$

$$P_B - P_A = (\rho_w - \rho_0)gL \ (2)$$

Substitute equation (1) for $P_B - P_A$ into (2) to obtain

$$\frac{1}{2}\rho_a v^2 = (\rho_w - \rho_0)gL \quad \text{or}$$

$$v = \sqrt{\frac{2gL(\rho_w - \rho_0)}{\rho_a}} = \sqrt{2(9.80 \text{ m/s}^2)(0.0500 \text{ m})\left(\frac{1000 - 750}{1.29}\right)}$$

$$v = \boxed{13.8 \text{ m/s}}$$