Chapter 18 Solutions

18.1 The resultant wave function has the form

$$y = 2A_0 \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

(a) $A = 2A_0 \cos\left(\frac{\phi}{2}\right) = 2(5.00) \cos\left[\frac{-(\pi/4)}{2}\right] = \boxed{9.24 \text{ m}}$

(b) $f = \frac{\omega}{2\pi} = \frac{1200\pi}{2\pi} = \boxed{600 \text{ Hz}}$

***18.2** We write the second wave function as

$$y_2 = A \sin(kx - \omega t + \phi)$$

$$y_2 = (0.0800 \text{ m}) \sin[2\pi(0.100x - 80.0t) + \phi]$$

Then

$$y_1 + y_2 = (0.0800 \text{ m}) \sin[2\pi(0.100x - 80.0t)]$$

$$+ (0.0800 \text{ m}) \sin[2\pi(0.100x - 80.0t) + \phi]$$

$$= 2(0.0800 \text{ m}) \cos\frac{\phi}{2} \sin\left[2\pi(0.100x - 80.0t) + \frac{\phi}{2}\right]$$

We require $2(0.0800 \text{ m}) \cos\frac{\phi}{2} = 0.0800\sqrt{3}$

$$\cos\frac{\phi}{2} = \frac{\sqrt{3}}{2}$$

$$\phi = 60.0° = \frac{\pi}{3}$$

Then the second wave function is

$$y_2 = (0.0800 \text{ m}) \sin\left[2\pi\left(0.100x - 80.0t + \frac{1}{6}\right)\right]$$

$$y_2 = \boxed{(0.0800 \text{ m}) \sin[2\pi(0.100x - 80.0t + 0.167)]}$$

18.3 Suppose the waves are sinusoidal. The sum is

$$(4.00 \text{ cm}) \sin(kx - \omega t) + (4.00 \text{ cm}) \sin(kx - \omega t + 90.0°)$$

$$2(4.00 \text{ cm}) \sin(kx - \omega t + 45.0°) \cos 45.0°$$

So the amplitude is $(8.00 \text{ cm}) \cos 45.0° = \boxed{5.66 \text{ cm}}$

18.4 $2A_0 \cos\left(\frac{\phi}{2}\right) = A_0$, so $\frac{\phi}{2} = \cos^{-1}\left(\frac{1}{2}\right) = 60.0° = \frac{\pi}{3}$

Thus, the phase difference is $\phi = 120° = \frac{2\pi}{3}$

This phase difference results if the time delay is $\frac{T}{3} = \frac{1}{3f} = \frac{\lambda}{3v}$

Time delay $= \frac{3.00 \text{ m}}{3(2.00 \text{ m/s})} = \boxed{0.500 \text{ s}}$

18.5 Waves reflecting from the near end travel 28.0 m (14.0 m down and 14.0 m back), while waves reflecting from the far end travel 66.0 m. The path difference for the two waves is:

$$\Delta r = 66.0 \text{ m} - 28.0 \text{ m} = 38.0 \text{ m}$$

Since $\lambda = \frac{v}{f}$, then $\frac{\Delta r}{\lambda} = \frac{(\Delta r)f}{v} = \frac{(38.0 \text{ m})(246 \text{ Hz})}{343 \text{ m/s}} = 27.254$

or, $\Delta r = 27.254\lambda$

The phase difference between the two reflected waves is then

$$\phi = 0.254(1 \text{ cycle}) = 0.254(2\pi \text{ rad}) = \boxed{91.3°}$$

18.6 (a) First we calculate the wavelength: $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{21.5 \text{ Hz}} = 16.0 \text{ m}$

Then we note that the path difference equals $9.00 \text{ m} - 1.00 \text{ m} = \boxed{\frac{1}{2}\lambda}$

Therefore, the receiver will record a minimum in sound intensity.

(b) If the receiver is located at point (x, y), then we must solve:

$$\sqrt{(x+5.00)^2+y^2}-\sqrt{(x-5.00)^2+y^2}=\frac{1}{2}\lambda$$

Then, $\sqrt{(x+5.00)^2+y^2}=\sqrt{(x-5.00)^2+y^2}+\frac{1}{2}\lambda$

Square both sides and simplify to get: $20.0x-\frac{\lambda^2}{4}=\lambda\sqrt{(x-5.00)^2+y^2}$

Upon squaring again, this reduces to:

$$400x^2-10.0\lambda^2x+\frac{\lambda^4}{16.0}=\lambda^2(x-5.00)^2+\lambda^2y^2$$

Substituting $\lambda = 16.0$ m, and reducing, we have:

$$\boxed{9.00x^2-16.0y^2=144} \quad \text{or} \quad \frac{x^2}{16.0}-\frac{y^2}{9.00}=1$$

(When plotted this yields a curve called a hyperbola.)

18.7 We suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\sqrt{L^2+d^2}-L$.

He hears a minimum when this is $(2n-1)\lambda/2$ with $n = 1, 2, 3, \ldots$

Then, $\sqrt{L^2+d^2}-L=(n-1/2)v/f$

$$\sqrt{L^2+d^2}=(n-1/2)v/f+L$$

$$L^2+d^2=(n-1/2)^2v^2/f^2+L^2+2(n-1/2)vL/f$$

$$L=\frac{d^2-(n-1/2)^2v^2/f^2}{2(n-1/2)v/f} \qquad n=1,2,3,\ldots$$

This will give us the answer to (b). The path difference starts from nearly zero when the man is very far away and increases to d when $L = 0$. The number of minima he hears is the greatest integer solution to $d \ge (n-1/2)\,v/f$

n = greatest integer $\le df/v + 1/2$

(a) $df/v+\frac{1}{2}=(4.00\text{ m})(200/\text{s})/330\text{ m/s}+\frac{1}{2}=2.92$

He hears $\boxed{\text{two}}$ minima.

(b) With $n = 1$,

$$L = \frac{d^2 - (1/2)^2 v^2/f^2}{2(1/2)v/f} = \frac{(4.00\text{ m})^2 - (330\text{ m/s})^2/4(200/\text{s})^2}{(330\text{ m/s})/200/\text{s}}$$

$$L = \boxed{9.28\text{ m}}$$

with $n = 2$

$$L = \frac{d^2 - (3/2)^2 v^2/f^2}{2(3/2)v/f} = \boxed{1.99\text{ m}}$$

18.8 Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r = \sqrt{L^2 + d^2} - L$.

He hears a minimum when

$$\Delta r = (2n - 1)\left(\frac{\lambda}{2}\right) \text{ with } n = 1, 2, 3, \ldots$$

Then, $\sqrt{L^2 + d^2} - L = (n - 1/2)(v/f)$

$$\sqrt{L^2 + d^2} = (n - 1/2)(v/f) + L$$

$$L^2 + d^2 = (n - 1/2)^2(v/f)^2 + 2(n - 1/2)(v/f)L + L^2 \qquad (1)$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference Δr starts from nearly zero when the man is very far away and increases to d when $L = 0$.

(a) The number of minima he hears is the greatest integer value for which $L \geq 0$. This is the same as the greatest integer solution to $d \geq (n - 1/2)(v/f)$, or

$$\boxed{\text{number of minima heard} = n_{\text{max}} = \text{greatest integer} \leq d(f/v) + 1/2}$$

(b) From Equation 1, the distances at which minima occur are given by

$$\boxed{L_n = \frac{d^2 - (n - 1/2)^2(v/f)^2}{2(n - 1/2)(v/f)} \text{ where } n = 1, 2, \ldots, n_{\text{max}}}$$

18.9 $y = (1.50\text{ m}) \sin(0.400x) \cos(200t) = 2A_0 \sin kx \cos \omega t$

Therefore,

$$k = \frac{2\pi}{\lambda} = 0.400\frac{\text{rad}}{\text{m}} \qquad \lambda = \frac{2\pi}{0.400\text{ rad/m}} = \boxed{15.7\text{ m}}$$

and $\omega = 2\pi f$, so $f = \dfrac{\omega}{2\pi} = \dfrac{200\text{ rad/s}}{2\pi\text{ rad}} = \boxed{31.8\text{ Hz}}$

The speed of waves in the medium is

$$v = \lambda f = \frac{\lambda}{2\pi}\,2\pi f = \frac{\omega}{k} = \frac{200\text{ rad/s}}{0.400\text{ rad/m}} = \boxed{500\text{ m/s}}$$

18.10 $y = 0.0300\text{ m}\cos\left(\frac{x}{2}\right)\cos(40t)$

(a) nodes occur where $y = 0$:

$$\frac{x}{2} = (2n+1)\frac{\pi}{2}$$

so $x = \boxed{(2n+1)\pi = \pi, 3\pi, 5\pi, \ldots}$

(b) $y_{max} = 0.0300\text{ m}\cos\left(\frac{0.400}{2}\right) = \boxed{0.0294\text{ m}}$

18.11 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{NN} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{2(800\text{ s}^{-1})} = 0.214\text{ m}$$

If the speakers vibrate in phase, the point halfway between them is an

antinode, at $\frac{1.25\text{ m}}{2} = 0.625$ m from either speaker.

Then there is a node at

$0.625\text{ m} - \frac{0.214\text{ m}}{2} = \boxed{0.518\text{ m}}$, a node at

$0.518\text{ m} - 0.214\text{ m} = \boxed{0.303\text{ m}}$, a node at

$0.303\text{ m} - 0.214\text{ m} = \boxed{0.0891\text{ m}}$, a node at

$0.518\text{ m} + 0.214\text{ m} = \boxed{0.732\text{ m}}$, a node at

$0.732\text{ m} + 0.214\text{ m} = \boxed{0.947\text{ m}}$, and a node at

$0.947\text{ m} + 0.214\text{ m} = \boxed{1.16\text{ m}}$ from either speaker

***18.12** (a) The resultant wave is

$$y = 2A \sin\left(kx + \frac{\phi}{2}\right) \cos\left(\omega t - \frac{\phi}{2}\right)$$

The nodes are located at

$$kx + \frac{\phi}{2} = n\pi$$

so $$x = \frac{n\pi}{k} - \frac{\phi}{2k}$$

which means that each node is shifted $\frac{\phi}{2k}$ to the left.

(b) The separation of nodes is

$$\Delta x = \left[(n+1)\frac{\pi}{k} - \frac{\phi}{2k}\right] - \left[\frac{n\pi}{k} - \frac{\phi}{2k}\right]$$

$$\Delta x = \frac{\pi}{k} = \frac{\lambda}{2}$$

The nodes are still separated by half a wavelength.

18.13 $y_1 = 3.00 \sin [\pi(x + 0.600t)]$ cm $\qquad y_2 = 3.00 \sin [\pi(x - 0.600t)]$ cm

$y = y_1 + y_2 = [3.00 \sin (\pi x) \cos (0.600\,\pi t) + 3.00 \sin (\pi x) \cos (0.600\pi t)]$ cm

$= (6.00 \text{ cm}) \sin (\pi x) \cos (0.600\pi t)$

(a) We can take $\cos(0.600\pi t) = 1$ to get the maximum y.

At $x = 0.250$ cm, $y_{\text{max}} = (6.00 \text{ cm}) \sin(0.250\pi) = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500$ cm, $y_{\text{max}} = (6.00 \text{ cm}) \sin(0.500\pi) = \boxed{6.00 \text{ cm}}$

(c) Now take $\cos (0.600\pi t) = -1$ to get y_{max}:

At $x = 1.50$ cm, $y_{max} = (6.00 \text{ cm}) \sin(1.50\pi)(-1) = \boxed{6.00 \text{ cm}}$

(d) The antinodes occur when $x = n\lambda/4$ $(n = 1, 3, 5, \ldots)$. But

$k = 2\pi/\lambda = \pi$, so $\lambda = 2.00$ cm, and

$x_1 = \lambda/4 = \boxed{0.500 \text{ cm}}$ as in (b)

$x_2 = 3\lambda/4 = \boxed{1.50 \text{ cm}}$ as in (c)

$x_3 = 5\lambda/4 = \boxed{2.50 \text{ cm}}$

18.14 (a) Using the given parameters, the wave function is

$$y = 2\pi \sin\left(\frac{\pi x}{2}\right) \cos(10\pi t)$$

We need to find values of x for which $\left|\sin\left(\frac{\pi x}{2}\right)\right| = 1$

This condition requires that $\frac{\pi x}{2} = \pi\left(n + \frac{1}{2}\right)$; $n = 0, 1, 2, \ldots$

For $n = 0$, $x = 1.00$ cm and for $n = 1$, $x = 3.00$ cm

Therefore, the distance between antinodes, $\Delta x = \boxed{2.00 \text{ cm}}$

(b) $A = 2\pi \sin\left(\frac{\pi x}{2}\right)$; when $x = 0.250$ cm, $A = \boxed{2.40 \text{ cm}}$

18.15 $y = 2A_0 \sin kx \cos \omega t$

$$\frac{\partial^2 y}{\partial x^2} = -2A_0 k^2 \sin kx \cos \omega t$$

$$\frac{\partial^2 y}{\partial t^2} = -2A_0 \omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives

$$-2A_0 k^2 \sin kx \cos \omega t = \left(\frac{1}{v^2}\right)(-2A_0 \omega^2 \sin kx \cos \omega t)$$

This is satisfied, provided that $v = \frac{\omega}{k}$

18.16 $\mu = \frac{0.100 \text{ kg}}{2.00 \text{ m}} = 0.0500 \text{ kg/m}$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(20.0 \text{ kg} \cdot \text{m/s}^2)}{0.0500 \text{ kg/m}}} = 20.0 \text{ m/s}$$

For the simplest vibration possibility, NAN,

$$d_{NN} = 2.00 \text{ m} = \frac{\lambda}{2} \qquad \lambda = 4.00 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{(20.0 \text{ m/s})}{4.00 \text{ m}} = \boxed{5.00 \text{ Hz}}$$

For the second state NANAN,

$$d_{NN} = 1.00\text{ m} \qquad \lambda = 2.00\text{ m}$$

$$f = \frac{(20.0\text{ m/s})}{2.00\text{ m}} = \boxed{10.0\text{ Hz}}$$

For the third resonance, NANANAN,

$$d_{NN} = \frac{2.00\text{ m}}{3} \qquad \lambda = 1.33\text{ m} \qquad f = \boxed{15.0\text{ Hz}}$$

The mode mentioned in the problem has

$$d_{NN} = 0.400\text{ m} \qquad \lambda = 0.800\text{ m} \qquad f = \boxed{25.0\text{ Hz}}$$

It is the $\boxed{\text{fifth allowed state}}$.

18.17 $L = 30.0\text{ m} \qquad \mu = 9.00 \times 10^{-3}\text{ kg/m} \qquad T = 20.0\text{ N}$

$$f_1 = \frac{v}{2L}$$

where $v = \left(\frac{T}{\mu}\right)^{1/2} = 47.1\text{ m/s}$

so $f_1 = \frac{47.1}{60.0} = \boxed{0.786\text{ Hz}}$

$$f_2 = 2f_1 = \boxed{1.57\text{ Hz}} \qquad f_3 = 3f_1 = \boxed{2.36\text{ Hz}} \qquad f_4 = 4f_1 = \boxed{3.14\text{ Hz}}$$

<u>**Goal Solution**</u>

G: The string described in the problem is very long, loose, and somewhat massive, so it should have a very low fundamental frequency, maybe only a few vibrations per second.

O: The tension and linear density of the string can be used to find the wave speed, which can then be used along with the required wavelength to find the fundamental frequency.

A: The wave speed is $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{20\text{ N}}{9.0 \times 10^{-3}\text{ kg/m}}} = 47.1\text{ m/s}$

For a vibrating string of length L fixed at both ends, the wavelength of the fundamental frequency is $\lambda = 2L = 60.0$ m; and the frequency is

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L} = \frac{47.1\text{ m/s}}{60\text{ m}} = \boxed{0.786\text{ Hz}}$$

The next three harmonics are

$$f_2 = 2f_1 = \boxed{1.57 \text{ Hz}}$$

$$f_3 = 3f_1 = \boxed{2.36 \text{ Hz}}$$

$$f_4 = 4f_1 = \boxed{3.14 \text{ Hz}}$$

L: The fundamental frequency is even lower than expected, less than 1 Hz. In fact, all 4 of the lowest resonant frequencies are below the normal human hearing range (20 to 17 000 Hz), so these harmonics are not even audible.

18.18 $L = 120$ cm $\quad f = 120$ Hz

(a) For four segments,

$$L = 2\lambda$$

or $\lambda = 60.0 \text{ cm} = \boxed{0.600 \text{ m}}$

(b) $v = \lambda f = 72.0$ m/s

$$f_1 = \frac{v}{2L} = \frac{72.0}{2(1.20)} = \boxed{30.0 \text{ Hz}}$$

18.19 $d_{NN} = 0.700$ m

$\lambda = 1.40$ m

$$f\lambda = v = 308 \text{ m/s} = \sqrt{\frac{T}{(1.20 \times 10^{-3})/(0.700)}}$$

(a) $T = \boxed{163 \text{ N}}$

(b) $f_3 = \boxed{660 \text{ Hz}}$

Goal Solution

G: The tension should be less than 100 lbs. (~500 N) since excessive force on the 4 cello strings would break the neck of the instrument. If the string vibrates in three segments, there will be three antinodes (instead of one for the fundamental mode), so the frequency should be three times greater than the fundamental.

O: The length of the string can be used to find the wavelength, which can be used with the fundamental frequency to find the wave speed. The tension can then be found from the wave speed and linear mass density of the string.

A: When the string vibrates in the lowest frequency mode, the length of string forms a standing wave where $L = \lambda/2$ (see Figure 18.2b), so the fundamental harmonic wavelength is

$$\lambda = 2L = 2(0.700\text{ m}) = 1.40\text{ m}$$

and the velocity is $v = f\lambda = (220\text{ s}^{-1})(1.40\text{ m}) = 308\text{ m/s}$

From the tension equation $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}}$ we get

(a) $T = \frac{v^2 m}{L} = \frac{(308\text{ m/s})^2(1.20 \times 10^{-3}\text{ kg})}{0.700\text{ m}} = 163\text{ N}$

(b) For the third harmonic, the tension, linear density, and speed are the same. However, the string vibrates in three segments so that the wavelength is one third as long as in the fundamental (see Figure 18.2d).

$$\lambda_3 = \lambda/3$$

From the equation $\lambda f = v$, we find that the frequency is three times as high:

$$f_3 = \frac{v}{\lambda_3} = 3\frac{v}{\lambda} = 3f = 660\text{ Hz}$$

L: The tension seems reasonable, and the third harmonic is three times the fundamental frequency as expected. Related to part (b), some stringed instrument players use a technique to double the frequency of a note by "cutting" a vibrating string in half. When the string is suddenly held at its midpoint to form a node, the second harmonic is formed, and the resulting note is one octave higher (twice the original fundamental frequency).

***18.20** $f_1 = \frac{v}{2L}$, where $v = \left(\frac{T}{\mu}\right)^{1/2}$

(a) If L is doubled, then $f_1 \propto L^{-1}$ will be reduced by a factor $\frac{1}{2}$.

(b) If μ is doubled, then $f_1 \propto \mu^{-1/2}$ will be reduced by a factor $\frac{1}{\sqrt{2}}$.

(c) If T is doubled, then $f_1 \propto \sqrt{T}$ will increase by a factor of $\sqrt{2}$.

18.21 $L = 60.0\text{ cm} = 0.600\text{ m}$ $\quad T = 50.0\text{ N}$ $\quad \mu = 0.100\text{ g/cm} = 0.0100\text{ kg/m}$

$$f_n = \frac{nv}{2L}$$

where

$$v = \left(\frac{T}{\mu}\right)^{1/2} = 70.7\text{ m/s}$$

$$f_n = n\left(\frac{70.7}{1.20}\right) = 58.9n = 20{,}000\text{ Hz}$$

Largest $n = 339 \Rightarrow f = \boxed{19.976\text{ kHz}}$

18.22 $f = \frac{v}{\lambda} = \sqrt{\frac{T}{\mu}}\ \frac{1}{\lambda} = \sqrt{\frac{T4}{\rho\pi d^2}}\ \frac{2}{L}$

since $\mu = \frac{M}{L} = \frac{\rho V}{L} = \rho\frac{AL}{L}$

$$f_{\text{new}} = \sqrt{\frac{4T_{\text{old}}4}{\rho_{\text{old}}\pi(2d_{\text{old}})^2}}\ \frac{2}{L_{\text{old}}/2}$$

$$= \sqrt{\frac{T_{\text{old}}4}{\rho_{\text{old}}\ \pi\ d_{\text{old}}^2}}\ \frac{2}{L_{\text{old}}} \times 2 = 2f_{\text{old}} = \boxed{800\text{ Hz}}$$

18.23 $\lambda_G = 2(0.350\text{ m}) = \frac{v}{f_G}$

$$\lambda_A = 2L_A = \frac{v}{f_A}$$

$L_G - L_A = L_G - \left(\frac{f_G}{f_A}\right) L_G = L_G$ **Error!**) $= (0.350\text{ m})$ **Error!** $= 0.0382\text{ m}$

Thus, $L_A = L_G - 0.0382$ m $= 0.350$ m $- 0.0382$ m $= 0.312$ m, or the finger should be placed $\boxed{\text{31.2 cm from the bridge}}$.

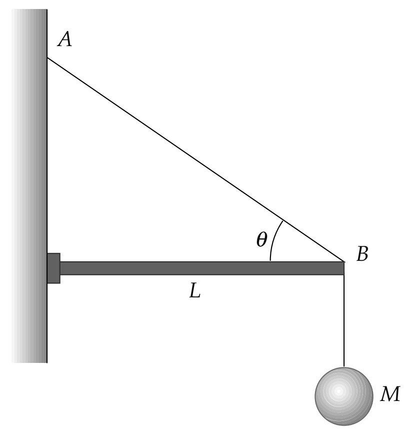
$$L_A = \frac{v}{2f_A} = \frac{1}{2f_A}\sqrt{\frac{T}{\mu}}$$

$$dL_A = \frac{dT}{4f_A\sqrt{T\mu}}$$

$$\frac{dL_A}{L_A} = \frac{1}{2}\frac{dT}{T}$$

$$\frac{dT}{T} = 2\frac{dL_A}{L_A} = 2\frac{0.600\text{ cm}}{(35.0 - 3.82)\text{ cm}} = \boxed{3.84\%}$$

18.24 In the fundamental mode, the string above the rod has only two nodes, at A and B, with an anti-node halfway between A and B. Thus,

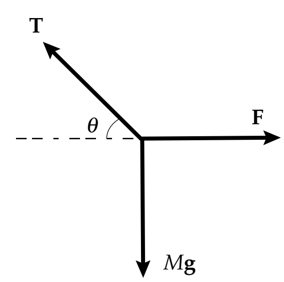


$$\frac{\lambda}{2} = \overline{AB} = \frac{L}{\cos\theta} \quad \text{or} \quad \lambda = \frac{2L}{\cos\theta}$$

Since the fundamental frequency is f, the wave speed in this segment of string is

$$v = \lambda f = \frac{2Lf}{\cos\theta}$$

Also, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/\overline{AB}}} = \sqrt{\frac{TL}{m\cos\theta}}$



where T is the tension in this part of the string. Thus,

$$\frac{2Lf}{\cos\theta} = \sqrt{\frac{TL}{m\cos\theta}} \quad \text{or} \quad \frac{4L^2f^2}{\cos^2\theta} = \frac{TL}{m\cos\theta}$$

and the mass of string above the rod is:

$$m = \frac{T\cos\theta}{4Lf^2} \qquad \text{[Equation 1]}$$

Now, consider the tension in the string. The light rod would rotate about point P if the string exerted any vertical force on it. Therefore, recalling Newton's third law, the rod must exert only a horizontal force on the string. Consider a free-body diagram of the string segment in contact with the end of the rod.

$$\Sigma F_y = T\sin\theta - Mg = 0 \Rightarrow T = \frac{Mg}{\sin\theta}$$

Then, from Equation 1, the mass of string above the rod is

$$m = \left(\frac{Mg}{\sin\theta}\right)\frac{\cos\theta}{4Lf^2} = \boxed{\frac{Mg}{4Lf^2\tan\theta}}$$

18.25 (a) Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n+1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = 2L/n$, and the frequency is $f = v/\lambda$.

Thus, $f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$, and also $f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}}$

Thus, $\frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0\text{ kg})g}{(16.0\text{ kg})g}} = \frac{5}{4}$

Therefore, $4n + 4 = 5n$, or $n = 4$

Then, $f = \frac{4}{2(2.00\text{ m})}\sqrt{\frac{(25.0\text{ kg})(9.80\text{ m/s}^2)}{0.00200\text{ kg/m}}} = \boxed{350\text{ Hz}}$

(b) The largest mass will correspond to a standing wave of 1 loop

$(n = 1)$, so $350\text{ Hz} = \frac{1}{2(2.00\text{ m})}\sqrt{\frac{m(9.80\text{ m/s}^2)}{0.00200\text{ kg/m}}}$

yielding $m = \boxed{400\text{ kg}}$

***18.26** Using the frets does not change the speed of the wave. Therefore, if d_{NN} is the distance between adjacent nodes,

$\lambda_1 f_1 = \lambda_2 f_2 = 2d_{NN1}f_1 = 2d_{NN2}f_2$ or

$$d_{NN2} = d_{NN1}\left(\frac{f_1}{f_2}\right) = 21.4\text{ cm}\left(\frac{2349\text{ Hz}}{2217\text{ Hz}}\right) = 22.7\text{ cm}$$

Thus, the distance between frets is

$$d_{NN2} - d_{NN1} = 22.7\text{ cm} - 21.4\text{ cm} = \boxed{1.27\text{ cm}}$$

***18.27** The natural frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{2.00\text{ m}}} = 0.352\text{ Hz}$$

The big brother must push at this same frequency of $\boxed{0.352\text{ Hz}}$ to produce resonance.

***18.28** The distance between adjacent nodes is one-quarter of the circumference.

$$d_{NN} = d_{AA} = \frac{\lambda}{2} = \frac{20.0\text{ cm}}{4} = 5.00\text{ cm}$$

so $\lambda = 10.0\text{ cm}$ and $f = \frac{v}{\lambda} = \frac{900\text{ m/s}}{0.100\text{ m}} = 9000\text{ Hz} = \boxed{9.00\text{ kHz}}$

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

***18.29** (a) The wave speed is $v = \frac{9.15\text{ m}}{2.50\text{ s}} = \boxed{3.66\text{ m/s}}$

(b) From Figure P18.29, there are antinodes at both ends, so the distance between adjacent antinodes is

$$d_{AA} = \frac{\lambda}{2} = 9.15\text{ m, and the wavelength is } \lambda = 18.3\text{ m}$$

The frequency is then $f = \frac{v}{\lambda} = \frac{3.66\text{ m/s}}{18.3\text{ m}} = \boxed{0.200\text{ Hz}}$

We have assumed the wave speed is the same for all wavelengths.

***18.30** The wave speed is $v = \sqrt{gd} = \sqrt{(9.80\text{ m/s}^2)(36.1\text{ m})} = 18.8\text{ m/s}$

The bay has one end open and one end closed, so its simplest resonance is with a node (of velocity, antinode of displacement) at the head of the bay and an antinode (of velocity, node of displacement) at the mouth. Then,

$$d_{NA} = 210 \times 10^3\text{ m} = \frac{\lambda}{4} \quad \text{and} \quad \lambda = 840 \times 10^3\text{ m}$$

Therefore, the period is

$$T = \frac{1}{f} = \frac{\lambda}{v} = \frac{840 \times 10^3\text{ m}}{18.8\text{ m/s}} = 4.47 \times 10^4\text{ s} = \boxed{12\text{ h }24\text{ min}}$$

$\boxed{\text{This agrees precisely with the period of the lunar excitation}}$, so we identify the extra-high tides as amplified by resonance.

18.31 (a) For the fundamental mode in a closed pipe, $\lambda = 4L$. (see Figure 18.3b)

But $v = f\lambda$, therefore $L = \frac{v}{4f}$

So, $L = \frac{343\text{ m/s}}{4(240\text{/s})} = \boxed{0.357\text{ m}}$

(b) For an open pipe, $\lambda = 2L$. (see Figure 18.3a)

So, $L = \frac{v}{2f} = \frac{343\text{ m/s}}{2(240\text{/s})} = \boxed{0.715\text{ m}}$

***18.32** $\dfrac{\lambda}{2} = d_{AA} = \dfrac{L}{n}$ or $L = \dfrac{n\lambda}{2}$ for $n = 1, 2, 3, \ldots$

Since $\lambda = \dfrac{v}{f}$, $L = n\left(\dfrac{v}{2f}\right)$ for $n = 1, 2, 3, \ldots$

With $v = 343$ m/s, and $f = 680$ Hz,

$$L = n\left(\frac{343 \text{ m/s}}{2(680 \text{ Hz})}\right) = n(0.252 \text{ m}) \text{ for } n = 1, 2, 3, \ldots$$

Possible lengths for resonance are:

$$L = \boxed{0.252 \text{ m}, 0.504 \text{ m}, 0.757 \text{ m}, \ldots, n(0.252) \text{ m}}$$

18.33 $d_{AA} = 0.320$ m $\lambda = 0.640$ m

(a) $f = \dfrac{v}{\lambda} = \boxed{531 \text{ Hz}}$

(b) $\lambda = 0.0850$ m $d_{AA} = \boxed{42.5 \text{ mm}}$

***18.34** The wavelength is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{261.6/\text{s}} = 1.31 \text{ m}$$

so the length of the open pipe vibrating in its simplest (A-N-A) mode is

$$d_{A \text{ to } A} = \frac{1}{2}\lambda = \boxed{0.656 \text{ m}}$$

A closed pipe has (N-A) for its simplest resonance, (N-A-N-A) for the second, and (N-A-N-A-N-A) for the third. Here, the pipe length is

$$5d_{N \text{ to } A} = \frac{5\lambda}{4} = \frac{5}{4}(1.31 \text{ m}) = \boxed{1.64 \text{ m}}$$

***18.35** The air in the auditory canal, about 3 cm long, can vibrate with a node at the closed end and antinode at the open end, with

$$d_{N \text{ to } A} = 3 \text{ cm} = \frac{\lambda}{4}$$

so $\lambda = 0.12$ m

and $f = \dfrac{v}{\lambda} = \dfrac{343 \text{ m/s}}{0.12 \text{ m}} \approx \boxed{3 \text{ kHz}}$

A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

18.36 $\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{440/\text{s}} = 0.780\text{ m}$

$d_{N\text{ to }A} = \frac{\lambda}{4} = 0.195\text{ m}$ = length of resonant air column

Water height = 0.400 m – 0.195 m = 0.205 m

$m = \rho V = \rho A h = (1000\text{ kg/m}^3)(0.100\text{ m}^2)(0.205\text{ m}) = \boxed{20.5\text{ kg}}$

18.37 For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $L = \frac{n\lambda}{2}$, $(n = 1, 2, 3, \ldots)$.

i.e., $L = \frac{n\lambda}{2} = \frac{nv}{2f}$

and $f = \frac{nv}{2L}$

Therefore, with $L = 0.860$ m and $L' = 2.10$ m, the resonant frequencies are

$f_n = \boxed{n(206\text{ Hz})}$ for $L = 0.860$ m for each n from 1 to 9

and $f'_n = \boxed{n(84.5\text{ Hz})}$ for $L' = 2.10$ m for each n from 2 to 23

18.38 We suppose these are the lowest resonances of the enclosed air columns.

For one,

$$\lambda = \frac{v}{f} = \frac{(343\text{ m/s})}{256/\text{s}} = 1.34\text{ m}$$

$$\text{length} = d_{AA} = \frac{\lambda}{2} = 0.670\text{ m}$$

For the other,

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{440/\text{s}} = 0.780\text{ m}$$

length = 0.390 m

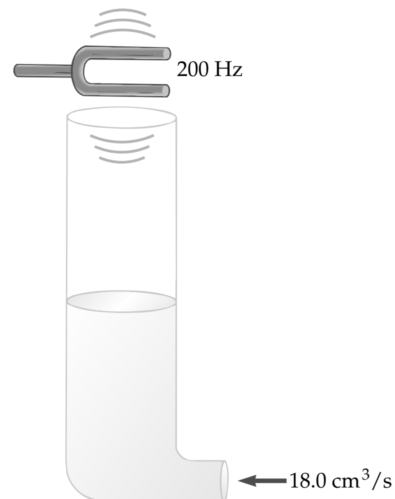
So,

(b) original length = $\boxed{1.06\text{ m}}$

$$\lambda = 2d_{AA} = 2.12\text{ m}$$

(a) $f = \frac{(343\text{ m/s})}{2.12\text{ m}} = \boxed{162\text{ Hz}}$

18.39 The fork radiates sound with $\lambda = \frac{v}{f}$



The distance between successive water levels at resonance is

$$d_{NN} = \frac{v}{2f}$$

So $$Rt = \frac{\pi r^2 v}{2f}$$

$$t = \frac{\pi r^2 v}{2Rf}$$

$$t = \frac{\pi(4.00 \times 10^{-2}\text{ m})^2(343\text{ m/s})}{2(18.0 \times 10^{-6}\text{ m}^3/\text{s})(200/\text{s})} = \boxed{239\text{ s}}$$

18.40 The wavelength of sound is $\lambda = \frac{v}{f}$

The distance between water levels at resonance is

$$d = \frac{v}{2f}$$

$$\therefore Rt = \pi r^2 d = \frac{\pi r^2 v}{2f}$$

and $$t = \boxed{\frac{\pi r^2 v}{2Rf}}$$

18.41 The length corresponding to the fundamental satisfies $f = \frac{v}{4L}$, giving

$$L_1 = \frac{v}{4f} = \frac{343}{4(512)} = 0.167\text{ m}$$

Since $L > 20.0$ cm, the *next* two modes will be observed, corresponding to

$$f = \frac{3v}{4L_2} \quad \text{and} \quad f = \frac{5v}{4L_3}$$

or $$L_2 = \frac{3v}{4f} = \boxed{0.502\text{ m}} \quad \text{and} \quad L_3 \frac{5v}{4f} = \boxed{0.837\text{ m}}$$

18.42 Call L the depth of the well and v the speed of sound. Then for some integer n

$$L = (2n-1)\frac{\lambda_1}{4} = (2n-1)\frac{v}{4f_1} = \frac{(2n-1)(343 \text{ m/s})}{4(51.5 \text{ s}^{-1})}$$

and for the next resonance

$$L = [2(n+1)-1]\frac{\lambda_2}{4} = (2n+1)\frac{v}{4f_2} = \frac{(2n+1)(343 \text{ m/s})}{4(60.0 \text{ s}^{-1})}$$

Thus, $\frac{(2n-1)(343 \text{ m/s})}{4(51.5 \text{ s}^{-1})} = \frac{(2n+1)(343 \text{ m/s})}{4(60.0 \text{ s}^{-1})}$,

and we require an *integer* solution to $\frac{2n+1}{60.0} = \frac{2n-1}{51.5}$

The equation gives $n = \frac{111.5}{17} = 6.56$, so the best fitting integer is $n = 7$.

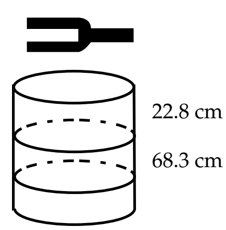
Then $L = \frac{[2(7)-1](343 \text{ m/s})}{4(51.5 \text{ s}^{-1})} = 21.6 \text{ m}$

and $L = \frac{[2(7)+1](343 \text{ m/s})}{4(60.0 \text{ s}^{-1})} = 21.4 \text{ m}$

suggest the best value for the depth of the well is $\boxed{21.5 \text{ m}}$.

18.43 For resonance in a tube open at one end,

$$f = n\frac{v}{4L} \quad (n = 1, 3, 5, \ldots) \qquad \text{Equation 18.12}$$



(a) Assuming $n = 1$ and $n = 3$,

$$384 = \frac{v}{4(0.228)} \quad \text{and} \quad 384 = \frac{3v}{4(0.683)}$$

In either case, $v = \boxed{350 \text{ m/s}}$

(b) For the next resonance, $n = 5$, and

$$L = \frac{5v}{4f} = \frac{5(350 \text{ m/s})}{4(384 \text{ s}^{-1})} = \boxed{1.14 \text{ m}}$$

18.44 (a) $f_1 = \frac{v}{\lambda} = \frac{v}{4L} = \frac{331.5\text{ m/s}}{4(4.88\text{ m})} = \boxed{17.0\text{ Hz}}$

(b) $f_1 = \frac{v}{\lambda} = \frac{v}{2L} = \boxed{34.0\text{ Hz}}$

(c) For the closed pipe, $f = \frac{v(20.0°\text{C})}{v(0°\text{C})} f_1 = \sqrt{1 + \frac{20.0}{273}}\ f_1 = \boxed{17.6\text{ Hz}}$

For the open pipe, $f = \sqrt{1 + \frac{20.0}{273}}\ f_1 = \boxed{35.2\text{ Hz}}$

***18.45** (a) For the fundamental mode of an open tube,

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{2(880\text{ s}^{-1})} = \boxed{0.195\text{ m}}$$

(b) $v = 331\text{ m/s}\sqrt{1 + \frac{(-5.00)}{273}} = 328\text{ m/s}$

We ignore the thermal expansion of the metal.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{328\text{ m/s}}{2(0.195\text{ m})} = \boxed{841\text{ Hz}}$$

The flute is flat by a semitone.

18.46 When the rod is clamped at one-quarter of its length, the fundamental frequency corresponds to a mode of vibration in which $L = \lambda$.

Therefore, $L = \frac{v}{f} = \frac{5100\text{ m/s}}{4400\text{ Hz}} = \boxed{1.16\text{ m}}$

18.47 (a) $f = \frac{v}{2L} = \frac{5100}{(2)(1.60)} = \boxed{1.59\text{ kHz}}$

(b) Since it is held in the center, there must be a node in the center as well as antinodes at the ends. The even harmonics have an antinode at the center so only $\boxed{\text{the odd harmonics}}$ are present.

(c) $f = \frac{v'}{2L} = \frac{3560}{(2)(1.60)} = \boxed{1.11\text{ kHz}}$

18.48 $v = 4500\text{ m/s}$ $\lambda_1 = 4L = 240\text{ cm} = 2.40\text{ m}$

so, $f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = \frac{4500}{2.40} = \boxed{1.88\text{ kHz}}$

18.49 $f \propto v \propto \sqrt{T}$

$$f_{\text{new}} = 110\sqrt{\frac{540}{600}} = 104.4 \text{ Hz}$$

$$\Delta f = \boxed{5.64 \text{ beats/s}}$$

Goal Solution

G: Beat frequencies are usually only a few Hertz, so we should not expect a frequency much greater than this.

O: As in previous problems, the two wave speed equations can be used together to find the frequency of vibration that corresponds to a certain tension. The beat frequency is then just the difference in the two resulting frequencies from the two strings with different tensions.

A: Combining the velocity equation $v = f/\lambda$ and the tension equation $v = \sqrt{\frac{T}{\mu}}$ we find that

$$f = \sqrt{\frac{T}{\mu\lambda^2}}$$

and since μ and λ are constant, we can divide to get $\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}}$

With $f_1 = 110$ Hz, $T_1 = 600$ N, and $T_2 = 540$ N: $f_2 = (110 \text{ Hz})\sqrt{\frac{540 \text{ N}}{600 \text{ N}}} = 104.4 \text{ Hz}$

The beat frequency is: $f_b = \left|f_1 - f_2\right| = 110 \text{ Hz} - 104.4 \text{ Hz} = 5.64 \text{ Hz}$

L: As expected, the beat frequency is only a few cycles per second. This result from the interference of the two sound waves with slightly different frequencies has a tone that varies in amplitude over time, similar to the sound made by saying "wa-wa-wa…"
Note: The beat frequency above is written with three significant figures on the assumption that the data and known precisely enough to warrant them. This assumption implies that the original frequency is known more precisely than to the three significant digits quoted in "110 Hz." For example, if the original frequency of the strings were 109.6 Hz, the beat frequency would be 5.62 Hz.

***18.50** (a) The string could be tuned to either $\boxed{\text{521 Hz or 525 Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down. Instead, the frequency must have started at 525 Hz to become $\boxed{\text{526 Hz}}$.

(c) From $f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu}}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$,

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \quad \text{and} \quad T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1 = \left(\frac{523 \text{ Hz}}{526 \text{ Hz}}\right)^2 T_1 = 0.989 T_1$$

The fractional change that should be made in the tension is then

$$\text{fractional change} = \frac{T_1 - T_2}{T_1} = 1 - 0.989 = 0.0114 = 1.14\% \text{ lower}$$

The tension should be $\boxed{\text{reduced by } 1.14\%}$.

18.51 For an echo $f' = f\frac{(v + v_s)}{(v - v_s)}$

the beat frequency is $f_b = \boxed{|f' - f|}$

Solving for f_b gives $f_b = f\frac{(2v_s)}{(v - v_s)}$ when approaching wall.

(a) $f_b = (256)\frac{(2)(1.33)}{(343 - 1.33)} = \boxed{1.99 \text{ Hz}}$ beat frequency

(b) When moving away from wall, v_s changes sign. Solving for v_s gives

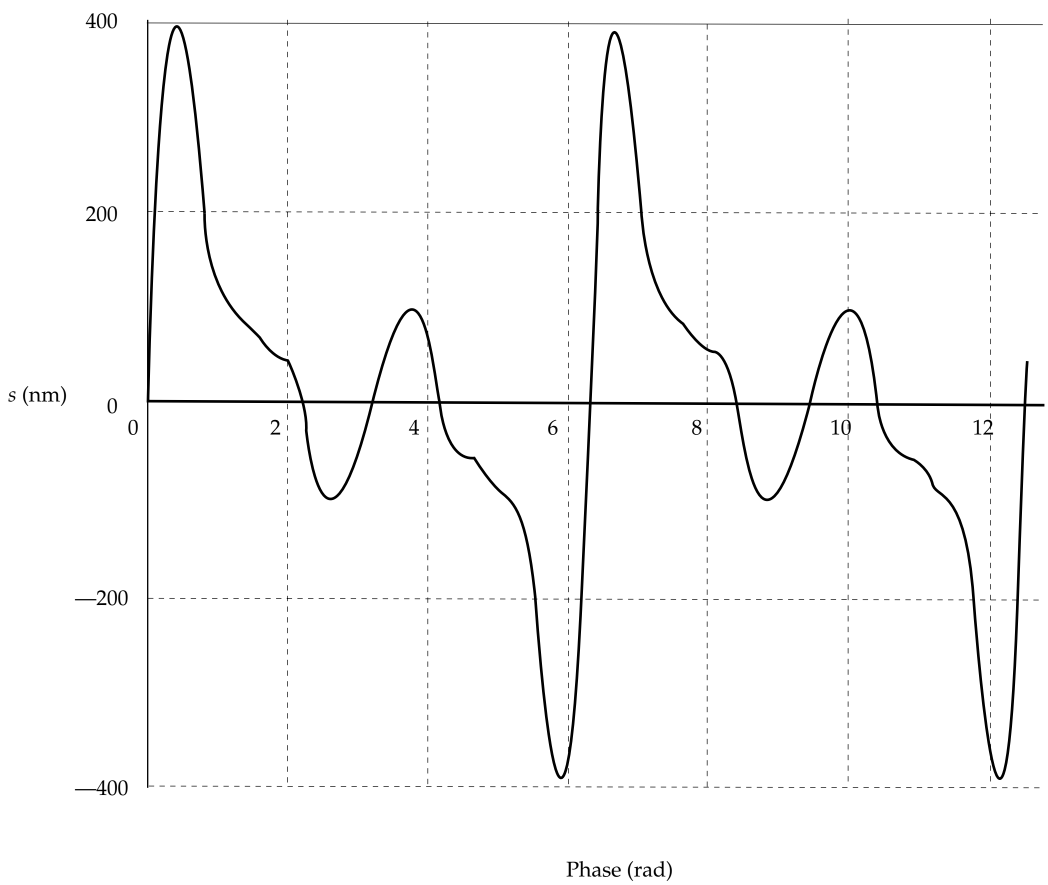
$$v_s = \frac{f_b v}{2f - f_b} = \frac{(5)(343)}{(2)(256) - 5} = \boxed{3.38 \text{ m/s}}$$

***18.52** We evaluate

$$s = 100 \sin \theta + 157 \sin 2\theta + 62.9 \sin 3\theta + 105 \sin 4\theta + 51.9 \sin 5\theta + 29.5 \sin 6\theta + 25.3 \sin 7\theta$$

where s represents particle displacement in nanometers and θ represents the phase of the wave in radians. As θ advances by 2π, time advances by (1/523) s. Here is the result:

Flute Waveform

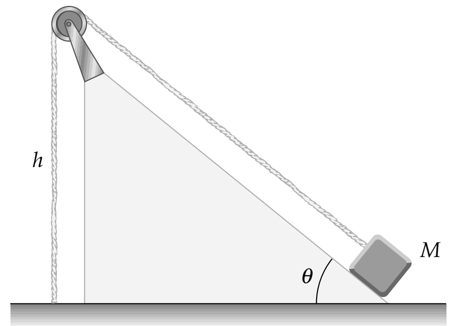


***18.53** We list the frequencies of the harmonics of each note in Hz:

	Harmonic				
Note	1	2	3	4	5
A	440.00	880.00	1320.0	1760.0	2200.0
C#	554.37	1108.7	1663.1	2217.5	2771.9
E	659.26	1318.5	1977.8	2637.0	3296.3

The second harmonic of E is close to the third harmonic of A, and the fourth harmonic of C# is close to the fifth harmonic of A.

18.54 (a) For the block:



$$\sum F_x = T - Mg\sin 30.0° = 0$$

so $T = Mg\sin 30.0° = \boxed{\frac{1}{2}Mg}$

(b) The length of the section of string parallel to the incline is $h/\sin 30.0° = 2h$. The total length of the string is then $\boxed{3h}$.

(c) The mass per unit length of the string is $\mu = \boxed{m/3h}$.

(d) The speed of waves in the string is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\left(\frac{Mg}{2}\right)\left(\frac{3h}{m}\right)} = \boxed{\sqrt{\frac{3Mgh}{2m}}}$

(e) In the fundamental mode, the segment of length h vibrates as one loop. The distance between adjacent nodes is then $d_{NN} = \lambda/2 = h$, so the wavelength is $\lambda = 2h$.

The frequency is $f = \frac{v}{\lambda} = \frac{1}{2h}\sqrt{\frac{3Mgh}{2m}} = \boxed{\sqrt{\frac{3Mg}{8mh}}}$

(g) When the vertical segment of string vibrates with 2 loops (i.e., 3 nodes), then $h = 2\left(\frac{\lambda}{2}\right)$ and the wavelength is $\lambda = \boxed{h}$.

(f) The period of the standing wave of 3 nodes (or two loops) is

$$T = \frac{1}{f} = \frac{\lambda}{v} = h\sqrt{\frac{2m}{3Mgh}} = \boxed{\sqrt{\frac{2mh}{3Mg}}}$$

(h) $f_b = 1.02f - f = (2.00 \times 10^{-2})f = \boxed{(2.00 \times 10^{-2})\sqrt{\frac{3Mg}{8mh}}}$

18.55 (a) $\Delta x = \sqrt{(9.00 + 4.00)} - 3.00 = \sqrt{13.0} - 3.00 = 0.606 \text{ m}$

The wavelength is $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{300 \text{ Hz}} = 1.14 \text{ m}$

Thus, $\frac{\Delta x}{\lambda} = \frac{0.606}{1.14} = 0.530$

of a wave, or

$$\Delta\phi = 2\pi(0.530) = \boxed{3.33 \text{ rad}}$$

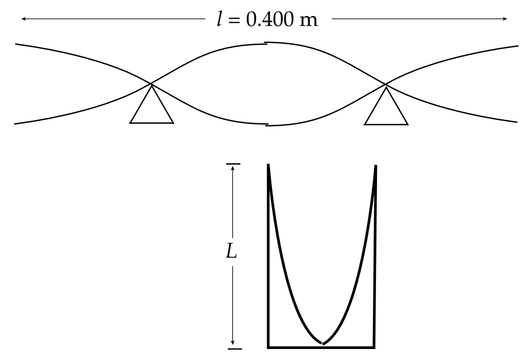
(b) For destructive interference, we want

$$\frac{\Delta x}{\lambda} = 0.500 = f\frac{\Delta x}{v}$$

where Δx is a constant in this set up.

$$f = \frac{v}{2\,\Delta x} = \frac{343}{(2)(0.606)} = \boxed{283\text{ Hz}}$$

18.56 $f = 87.0$ Hz



speed of sound in air: $v_a = 340$ m/s

(a) $\lambda_b = l$

$$v = f\lambda_b = (87.0\text{ s}^{-1})(0.400\text{ m})$$

$$v = \boxed{34.8\text{ m/s}}$$

(b) $$\left.\begin{array}{l}\lambda_a = 4L \\ v_a = \lambda_a f\end{array}\right\} \; L = \frac{v_a}{4f} = \frac{340\text{ m/s}}{4(87.0\text{ s}^{-1})} = \boxed{0.977\text{ m}}$$

18.57 Moving away from station, frequency is depressed:

$$f' = 180 - 2.00 = 178\text{ Hz}$$

$$178 = 180\frac{(343)}{(343 + v)}$$

Solving for v gives

$$v = \frac{(2.00)(343)}{178}$$

Therefore,

$$v = \boxed{3.85\text{ m/s away from station}}$$

Moving towards the station, the frequency is enhanced:

$$f' = 180 + 2.00 = 182\text{ Hz}$$

$$182 = 180\frac{(343)}{(343 - v)}$$

Solving for v gives

$$v = \frac{(2.00)(343)}{182}$$

Therefore,

$$v = \boxed{3.77 \text{ m/s towards the station}}$$

***18.58** Use the Doppler formula

$$f' = f\frac{(v \pm v_0)}{(v \mp v_s)}$$

With f_1' = frequency of the speaker in front of student and

f_2' = frequency of the speaker behind the student.

$$f_1' = (456 \text{ Hz})\frac{(343 \text{ m/s} + 1.50 \text{ m/s})}{(343 \text{ m/s} - 0)} = 458 \text{ Hz}$$

$$f_2' = (456 \text{ Hz})\frac{(343 \text{ m/s} - 1.50 \text{ m/s})}{(343 \text{ m/s} + 0)} = 454 \text{ Hz}$$

Therefore, $f_b = f_1' - f_1' = \boxed{3.99 \text{ Hz}}$

18.59 From the leading train she hears

$$f_1' = f\left(\frac{v + 0}{v + v_s}\right) = f\left(\frac{343 \text{ m/s}}{343 \text{ m/s} + 8.00 \text{ m/s}}\right)$$

From the still-approaching train,

$$f_2' = f\left(\frac{343}{343 - 8.00}\right)$$

Then, $4.00 \text{ Hz} = f_2' - f_1' = 1.0239f - 0.9772f$

$$f = \frac{4.00 \text{ Hz}}{0.0467} = \boxed{85.7 \text{ Hz}}$$

18.60 $v = \sqrt{\frac{(48.0)(2.00)}{4.80 \times 10^{-3}}} = 141 \text{ m/s}$

$d_{NN} = 1.00 \text{ m}$ $\lambda = 2.00 \text{ m}$ $f = \frac{v}{\lambda} = 70.7 \text{ Hz}$

$$\lambda_a = \frac{v_a}{f} = \frac{343 \text{ m/s}}{70.7 \text{ Hz}} = \boxed{4.85 \text{ m}}$$

***18.61** The second standing wave mode of the air in the pipe reads ANAN, with

$$d_{NA} = \frac{\lambda}{4} = \frac{1.75 \text{ m}}{3} \quad \text{so} \quad \lambda = 2.33 \text{ m} \text{ and}$$

$$f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{2.33 \text{ m}} = 147 \text{ Hz}$$

For the string, λ and v are different but f is the same.

$$\frac{\lambda}{2} = d_{NN} = \frac{0.400 \text{ m}}{2} \quad \text{so} \quad \lambda = 0.400 \text{ m}$$

$$v = \lambda f = (0.400 \text{ m})(147 \text{ Hz}) = 58.8 \text{ m/s} = \sqrt{T/\mu}$$

$$T = \mu v^2 = (9.00 \times 10^{-3} \text{ kg/m})(58.8 \text{ m/s})^2 = \boxed{31.1 \text{ N}}$$

18.62 (a) $L = \frac{v}{4f}$ so $\frac{L'}{L} = \frac{f}{f'}$

Letting the longest L be 1, the ratio is $1 : \frac{4}{5} : \frac{2}{3} : \frac{1}{2}$

or in integers $\boxed{30 : 24 : 20 : 15}$

(b) $L = \boxed{\frac{343}{(4)(256)} = 33.5 \text{ cm}}$

This is the longest pipe, so using the ratios the lengths are:

$\boxed{33.5, 26.8, 22.3, 16.7 \text{ cm}}$

(c) The frequencies are using the ratio $\boxed{256, 320, 384, \text{ and } 512 \text{ Hz}}$. These represent notes *C*, *E*, *G*, and *C′* on the physical pitch scale.

18.63 (a) Since the first node is at the weld, the wavelength in the thin wire is $2L$ or 80.0 cm. The frequency and tension are the same in both sections, so

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(0.400)}\sqrt{\frac{4.60}{2.00 \times 10^{-3}}} = \boxed{59.9 \text{ Hz}}$$

(b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire.

$\mu' = 8.00\ \text{g/m}$

so $L' = \frac{1}{2f}\sqrt{\frac{T}{\mu'}}$

$$L' = \left[\frac{1}{(2)(59.9)}\right]\sqrt{\left[\frac{(4.60)}{(8.00 \times 10^{-3})}\right]}$$

$= \boxed{20.0\ \text{cm}}$ half the length of the thin wire

18.64 $f_B = f_A$ $\lambda_B = \frac{1}{3}\lambda_A$ $v_B = \frac{1}{3}v_A$

$$v_B^2 = \frac{1}{9}v_A^2$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\frac{T_B}{T_A} = \frac{v_B^2}{v_A^2} = \boxed{0.111}$$

18.65 (a) $f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

so $\frac{f'}{f} = \frac{L}{L'} = \frac{L}{2L} = \frac{1}{2}$

The $\boxed{\text{frequency should be halved}}$ to get the same number of antinodes for twice the length.

(b) $\frac{n'}{n} = \sqrt{\frac{T}{T'}}$

so $\frac{T'}{T} = \left(\frac{n}{n'}\right)^2 = \left[\frac{n}{(n+1)}\right]^2$

The tension must be

$$T' = \boxed{\left[\frac{n}{(n+1)}\right]^2 T}$$

(c) $\frac{f'}{f} = \frac{n'}{n}\frac{L}{L'}\sqrt{\frac{T'}{T}}$

so $\frac{T'}{T} = \left(\frac{n}{n'}\frac{f'L'}{fL}\right)^2$

$\frac{T'}{T} = \left(\frac{3}{2 \cdot 2}\right)^2$

$\boxed{\frac{T'}{T} = \frac{9}{16}}$ to get twice as many antinodes.

18.66 For the wire,

$$\mu = \frac{0.0100 \text{ kg}}{2.00 \text{ m}} = 5.00 \times 10^{-3} \text{ kg/m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(200 \text{ kg} \cdot \text{m/s}^2)}{5.00 \times 10^{-3} \text{ kg/m}}}$$

$$v = 200 \text{ m/s}$$

If it vibrates in its simplest state,

$$d_{NN} = 2.00 \text{ m} = \frac{\lambda}{2}$$

$$f = \frac{v}{\lambda} = \frac{(200 \text{ m/s})}{4.00 \text{ m}} = 50.0 \text{ Hz}$$

(a) The tuning fork can have frequencies

$\boxed{45.0 \text{ Hz} \quad \text{or} \quad 55.0 \text{ Hz}}$

(b) If $f = 45.0$ Hz,

$$v = f\lambda = (45.0/\text{s})\ 4.00 \text{ m} = 180 \text{ m/s}$$

Then,

$$T = v^2\mu = (180 \text{ m/s})^2(5.00 \times 10^{-3} \text{ kg/m}) = \boxed{162 \text{ N}}$$

$\boxed{\text{or}}$ if $f = 55.0$ Hz

$$T = v^2\mu = f^{\,2}\lambda^2\mu = (55.0/\text{s})^2(4.00 \text{ m})^2(5.00 \times 10^{-3} \text{ kg/m}) = \boxed{242 \text{ N}}$$

18.67 The odd-numbered harmonics of the organ-pipe vibration are:

650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50.0 Hz

Closed $f_1 = \boxed{50.0\text{ Hz}}$ $\lambda = 6.80\text{ m}$ $\boxed{L = 1.70\text{ m}}$

18.68 We look for a solution of the form

$$5.00\sin(2.00x - 10.0t) + 10.0\cos(2.00x - 10.0t)$$

$$= A\sin(2.00x - 10.0t + \phi)$$

$$= A\sin(2.00x - 10.0t)\cos\phi + A\cos(2.00x - 10.0t)\sin\phi$$

This will be true if both $5.00 = A\cos\phi$ and $10.0 = A\sin\phi$,

requiring $(5.00)^2 + (10.0)^2 = A^2$ $A = 11.2$ and $\phi = 63.4°$

The resultant wave $\boxed{11.2\sin(2.00x - 10.0t + 63.4°)}$ is sinusoidal.

***18.69** (a) With $k =$ **Error!** and $\omega = 2\pi f =$ **Error!**

$$y(x, t) = 2A\sin kx\cos\omega t = \boxed{2A\sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi v t}{\lambda}\right)}$$

(b) For the fundamental vibration,

$$\lambda_1 = 2L$$

so $$y_1(x, t) = \boxed{2A\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi v t}{L}\right)}$$

(c) For the second harmonic

$$\lambda_2 = L$$

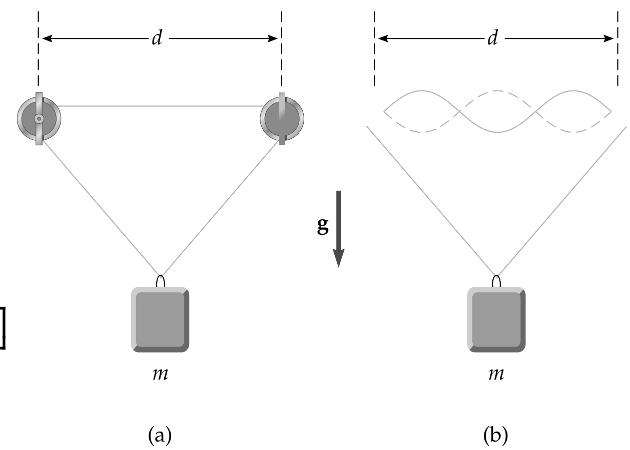
and

$$y_2(x, t) = \boxed{2A\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi v t}{L}\right)}$$

(d) In general,

$$\lambda_n = \frac{2L}{n} \quad \text{and} \quad y_n(x, t) = \boxed{2A\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi v t}{L}\right)}$$

18.70 (a) In the diagram, observe that:



$$\sin\theta = \frac{1.00\text{ m}}{1.50\text{ m}} = \frac{2}{3} \quad \text{or} \quad \theta = 41.8°$$

Considering the mass,

$$\Sigma F_y = 0 \quad \text{gives} \quad 2T\cos\theta = mg$$

or $$T = \frac{(12.0\text{ kg})(9.80\text{ m/s}^2)}{2\cos 41.8°} = \boxed{78.9\text{ N}}$$

(b) The speed of transverse waves in the string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{78.9\text{ N}}{0.00100\text{ kg/m}}} = 281\text{ m/s}$$

For the standing wave pattern shown (3 loops), $d = \frac{3}{2}\lambda$, or

$$\lambda = \frac{2(2.00\text{ m})}{3} = 1.33\text{ m}$$

Thus, the required frequency is

$$f = \frac{v}{\lambda} = \frac{281\text{ m/s}}{1.33\text{ m}} = \boxed{211\text{ Hz}}$$