Chapter 24 Solutions

24.1 (a) $\Phi_E = EA\cos\theta = (3.50 \times 10^3)(0.350 \times 0.700)\cos 0° = \boxed{858\ \text{N} \cdot \text{m}^2/\text{C}}$

(b) $\theta = 90.0°$ $\boxed{\Phi_E = 0}$

(c) $\Phi_E = (3.50 \times 10^3)(0.350 \times 0.700)\cos 40.0° = \boxed{657\ \text{N} \cdot \text{m}^2/\text{C}}$

24.2 $\Phi_E = EA\cos\theta = (2.00 \times 10^4\ \text{N/C})(18.0\ \text{m}^2)\cos 10.0° = \boxed{355\ \text{kN} \cdot \text{m}^2/\text{C}}$

24.3 $\Phi_E = EA\cos\theta$

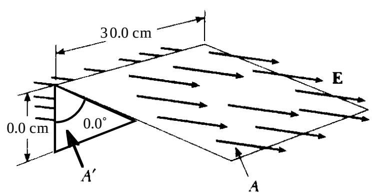
$A = \pi r^2 = \pi(0.200)^2 = 0.126\ \text{m}^2$

$5.20 \times 10^5 = E(0.126)\cos 0°$

$E = 4.14 \times 10^6\ \text{N/C} = \boxed{4.14\ \text{MN/C}}$

24.4 The uniform field enters the shell on one side and exits on the other so the total flux is $\boxed{\text{zero}}$.

24.5 (a) $A' = (10.0\ \text{cm})(30.0\ \text{cm})$



$A' = 300\ \text{cm}^2 = 0.0300\ \text{m}^2$

$\Phi_{E,A'} = EA'\cos\theta$

$\Phi_{E,A'} = (7.80 \times 10^4)(0.0300)\cos 180°$

$\Phi_{E,A'} = \boxed{-2.34\ \text{kN} \cdot \text{m}^2/\text{C}}$

(b) $\Phi_{E,A} = EA\cos\theta = (7.80 \times 10^4)(A)\cos 60.0°$

$$A = (30.0\ \text{cm})(w) = (30.0\ \text{cm})\left(\frac{10.0\ \text{cm}}{\cos 60.0°}\right) = 600\ \text{cm}^2 = 0.0600\ \text{m}^2$$

$\Phi_{E,A} = (7.80 \times 10^4)(0.0600)\cos 60° = \boxed{+2.34\ \text{kN} \cdot \text{m}^2/\text{C}}$

(c) The bottom and the two triangular sides all lie *parallel* to **E**, so $\Phi_E = 0$ for each of these. Thus,

$$\Phi_{E,\text{ total}} = -2.34\ \text{kN}\cdot\text{m}^2/\text{C} + 2.34\ \text{kN}\cdot\text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}$$

24.6 (a) $\Phi_E = \mathbf{E}\cdot\mathbf{A} = (a\,\mathbf{i} + b\,\mathbf{j})\cdot A\,\mathbf{i} = \boxed{aA}$

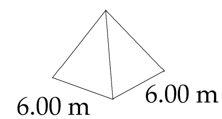
(b) $\Phi_E = (a\mathbf{i} + b\mathbf{j})\cdot A\mathbf{j} = \boxed{bA}$

(c) $\Phi_E = (a\mathbf{i} + b\mathbf{j})\cdot A\mathbf{k} = \boxed{0}$

24.7 Only the charge inside radius R contributes to the total flux.

$\Phi_E = \boxed{q / e_0}$

24.8 $\Phi_E = EA\cos\theta$ through the base



$\Phi_E = (52.0)(36.0)\cos 180° = -1.87 \text{ kN}\cdot\text{m}^2/\text{C}$

Note the same number of electric field lines go through the base as go through the pyramid's surface (not counting the base).

For the slanting surfaces, $\boxed{\Phi_E = +1.87 \text{ kN}\cdot\text{m}^2/\text{C}}$

24.9 The flux entering the closed surface equals the flux exiting the surface. The flux entering the left side of the cone is $\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = \boxed{ERh}$. This is the same as the flux that exits the right side of the cone. Note that for a uniform field only the cross sectional area matters, not shape.

***24.10** (a) $E = \dfrac{k_e Q}{r^2}$

$8.90\times 10^2 = \dfrac{(8.99\times 10^9)Q}{(0.750)^2}$, But Q is negative since $\mathbf{E}$ points inward.

$Q = -5.56\times 10^{-8}\text{ C} = \boxed{-55.6 \text{ nC}}$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution.

24.11 (a) $\Phi_E = \dfrac{q_{in}}{e_0} = \dfrac{(+5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C})}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -6.89\times 10^6\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{-6.89\ \text{MN}\cdot\text{m}^2/\text{C}}$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

24.12 $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0}$

Through S_1 $\Phi_E = \dfrac{-2Q+Q}{\epsilon_0} = \boxed{-\dfrac{Q}{\epsilon_0}}$

Through S_2 $\Phi_E = \dfrac{+Q-Q}{\epsilon_0} = \boxed{0}$

Through S_3 $\Phi_E = \dfrac{-2Q+Q-Q}{\epsilon_0} = \boxed{-\dfrac{2Q}{\epsilon_0}}$

Through S_4 $\Phi_E = \boxed{0}$

24.13 (a) One-half of the total flux created by the charge q goes through the plane. Thus,

$$\Phi_{E,\text{ plane}} = \frac{1}{2}\Phi_{E,\text{ total}} = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \boxed{\frac{q}{2\epsilon_0}}$$

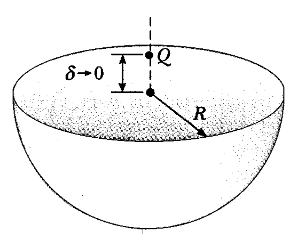
(b) The square looks like an infinite plane to a charge *very close* to the surface. Hence,

$$\Phi_{E,\text{ square}} \approx \Phi_{E,\text{ plane}} = \boxed{\frac{q}{2\epsilon_0}}$$

(c) $\boxed{\text{The plane and the square look the same to the charge.}}$

24.14 The flux through the curved surface is equal to the flux through the flat circle, $\boxed{E_0 \pi r^2}$.

24.15 (a) $\boxed{\dfrac{+Q}{2\epsilon_0}}$ Simply consider half of a closed sphere.



(b) $\boxed{\dfrac{-Q}{2\epsilon_0}}$ (from $\Phi_{E,\text{ total}} = \Phi_{E,\text{ dome}} + \Phi_{E,\text{ flat}} = 0$)

Goal Solution

A point charge Q is located just above the center of the flat face of a hemisphere of radius R, as shown in Figure P24.15. What is the electric flux (a) through the curved surface and (b) through the flat face?

G: From Gauss's law, the flux through a sphere with a point charge in it should be Q/ϵ_0, so we should expect the electric flux through a hemisphere to be half this value: $\Phi_{\text{curved}} = Q/2\epsilon_0$. Since the flat section appears like an infinite plane to a point just above its surface so that half of all the field lines from the point charge are intercepted by the flat surface, the flux through this section should also equal $Q/2\epsilon_0$.

O: We can apply the definition of electric flux directly for part (a) and then use Gauss's law to find the flux for part (b).

A: (a) With δ very small, all points on the hemisphere are nearly at distance R from the charge, so the field everywhere on the curved surface is $k_e Q / R^2$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:

$$\Phi_{\text{curved}} = \int \mathbf{E}\cdot d\mathbf{A} = E_{\text{local}}A_{\text{hemisphere}} = \left(k_e \frac{Q}{R^2}\right)\left(\tfrac{1}{2}\right)\left(4\pi R^2\right) = \frac{1}{4\pi\epsilon_0}Q(2\pi) = \frac{Q}{2\epsilon_0}$$

(b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \qquad \text{or} \qquad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \frac{-Q}{2\epsilon_0}$$

L: The direct calculations of the electric flux agree with our predictions, except for the negative sign in part (b), which comes from the fact that the area unit vector is defined as pointing outward from an enclosed surface, and in this case, the electric field has a component in the opposite direction (down).

24.16 (a) $\Phi_{E,\text{ shell}} = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{12.0\times10^{-6}}{8.85\times10^{-12}} = 1.36\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{1.36\ \text{MN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_{E,\text{ half shell}} = \tfrac{1}{2}(1.36\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C}) = 6.78\times10^{5}\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{678\ \text{kN}\cdot\text{m}^2/\text{C}}$

(c) $\boxed{\text{No,}}$ the same number of field lines will pass through each surface, no matter how the radius changes.

24.17 From Gauss's Law, $\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \dfrac{q_{in}}{\epsilon_0}$.

Thus, $\Phi_E = \dfrac{Q}{\epsilon_0} = \dfrac{0.0462\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{5.22\ \text{kN}\cdot\text{m}^2/\text{C}}$

24.18 If $R \le d$, the sphere encloses no charge and $\Phi_E = q_{\text{in}} / \epsilon_0 = \boxed{0}$

If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$

so $\Phi_E = \boxed{2\lambda \sqrt{R^2 - d^2} / \epsilon_0}$

24.19 The total charge is $Q - 6|q|$. The total outward flux from the cube is $\left(Q - 6|q|\right) / \epsilon_0$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q - 6|q|}{6\epsilon_0}}$$

$$(\Phi_E)_{\text{one face}} = \frac{Q - 6|q|}{6\epsilon_0} = \frac{(5.00 - 6.00) \times 10^{-6} \text{ C} \cdot \text{N} \cdot \text{m}^2}{6 \times 8.85 \times 10^{-12} \text{ C}^2} = \boxed{-18.8 \text{ kN} \cdot \text{m}^2/\text{C}}$$

24.20 The total charge is $Q - 6|q|$. The total outward flux from the cube is $\left(Q - 6|q|\right) / \epsilon_0$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q - 6|q|}{6\epsilon_0}}$$

24.21 When $R < d$, the cylinder contains no charge and $\Phi_E = \boxed{0}$.

When $R > d$, $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \boxed{\dfrac{\lambda L}{\epsilon_0}}$

24.22 $$\Phi_{E,\text{ hole}} = \mathbf{E} \cdot \mathbf{A}_{\text{hole}} = \left(\frac{k_e Q}{R^2}\right)\left(\pi r^2\right) = \left(\frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(10.0 \times 10^{-6} \text{ C}\right)}{(0.100 \text{ m})^2}\right)\pi\left(1.00 \times 10^{-3} \text{ m}\right)^2$$

$$\Phi_{E,\text{ hole}} = \boxed{28.2 \text{ N} \cdot \text{m}^2/\text{C}}$$

24.23 $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{170 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = 1.92 \times 10^7\ \ \text{N} \cdot \text{m}^2/\text{C}$

(a) $(\Phi_E)_{\text{one face}} = \frac{1}{6}\Phi_E = \frac{1.92 \times 10^7\ \ \text{N} \cdot \text{m}^2/\text{C}}{6}$

$(\Phi_E)_{\text{one face}} = \boxed{3.20\ \text{MN} \cdot \text{m}^2/\text{C}}$

(b) $\Phi_E = \boxed{19.2\ \text{MN} \cdot \text{m}^2/\text{C}}$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an unsymmetrical charge distribution. The sides of the cube nearer the charge would have more flux and the ones farther away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.

24.24 (a) $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0}$

$8.60 \times 10^4 = \frac{q_{in}}{8.85 \times 10^{-12}}$

$q_{in} = 7.61 \times 10^{-7}\ \text{C} = \boxed{761\ \text{nC}}$

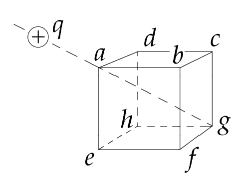
(b) Since the net flux is positive, $\boxed{\text{the net charge must be positive}}$. It can have any distribution.

(c) $\boxed{\text{The net charge would have the same magnitude but be negative.}}$

24.25 No charge is inside the cube. The net flux through the cube is zero. Positive flux comes out through the three faces meeting at *g*. These three faces together fill solid angle equal to one-eighth of a sphere as seen from *q*, and together pass flux $\frac{1}{8}(q/\epsilon_0)$. Each face containing *a* intercepts equal flux going into the cube:

$0 = \Phi_{E,\ \text{net}} = 3\Phi_{E,\ \text{abcd}} + q / 8\epsilon_0$

$\Phi_{E,\ \text{abcd}} = \boxed{-q / 24\epsilon_0}$



24.26 The charge distributed through the nucleus creates a field at the surface equal to that of a point charge at its center: $E = k_e q / r^2$

$$E = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(82 \times 1.60 \times 10^{-19}\text{ C})}{\left[(208)^{1/3}\ 1.20 \times 10^{-15}\text{ m}\right]^2}$$

$E = \boxed{2.33 \times 10^{21}\text{ N/C}}$ away from the nucleus

24.27 (a) $E = \dfrac{k_e Qr}{a^3} = \boxed{0}$

(b) $E = \dfrac{k_e Qr}{a^3} = \dfrac{(8.99 \times 10^9)(26.0 \times 10^{-6})(0.100)}{(0.400)^3} = \boxed{365\text{ kN/C}}$

(c) $E = \dfrac{k_e Q}{r^2} = \dfrac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.400)^2} = \boxed{1.46\text{ MN/C}}$

(d) $E = \dfrac{k_e Q}{r^2} = \dfrac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.600)^2} = \boxed{649\text{ kN/C}}$

The direction for each electric field is radially outward.

***24.28** (a) $E = \dfrac{2k_e \lambda}{r}$

$$3.60 \times 10^4 = \frac{2(8.99 \times 10^9)(Q/2.40)}{(0.190)}$$

$Q = +9.13 \times 10^{-7}\text{ C} = \boxed{+913\text{ nC}}$

(b) $\boxed{E = 0}$

24.29 $\oint \mathbf{E} \cdot d\mathbf{A} = \dfrac{q_{\text{in}}}{\text{e}_0} = \dfrac{\int \rho\, dV}{\text{e}_0} = \dfrac{\rho}{\text{e}_0}\, l \pi r^2$

$E 2\pi r l = \dfrac{\rho}{\text{e}_0}\, l \pi r^2$

$$\boxed{\mathbf{E} = \frac{\rho}{2\epsilon_0} r \text{ away from the axis}}$$

Goal Solution

Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where $r < R$.

G: According to Gauss's law, only the charge enclosed within the gaussian surface of radius r needs to be considered. The amount of charge within the gaussian surface will certainly increase as ρ and r increase, but the area of this gaussian surface will also increase, so it is difficult to predict which of these two competing factors will more strongly affect the electric field strength.

O: We can find the general equation for E from Gauss's law.

A: If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho\pi r^2 L$. The circular end caps have no electric flux through them; there $\mathbf{E}\cdot d\mathbf{A} = EdA\cos 90.0° = 0$. The curved surface has $\mathbf{E}\cdot d\mathbf{A} = EdA\cos 0°$, and E must be the same strength everywhere over the curved surface.

Gauss's law, $\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q}{\epsilon_0}$, becomes $E \int_{\text{Curved Surface}} dA = \frac{\rho\pi r^2 L}{\epsilon_0}$

Now the lateral surface area of the cylinder is $2\pi r L$: $E(2\pi r)L = \frac{\rho\pi r^2 L}{\epsilon_0}$

Thus, $\mathbf{E} = \frac{\rho r}{2\epsilon_0}$ radially away from the cylinder axis

L: As we expected, the electric field will increase as ρ increases, and we can now see that E is also proportional to r. For the region outside the cylinder ($r > R$), we should expect the electric field to decrease as r increases, just like for a line of charge.

24.30 $\sigma = \left(8.60\times10^{-6}\ \text{C/cm}^2\right)\left(\frac{100\ \text{cm}}{\text{m}}\right)^2 = 8.60\times10^{-2}\ \text{C/m}^2$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{8.60\times10^{-2}}{2\left(8.85\times10^{-12}\right)} = \boxed{4.86\times10^9\ \text{N/C}}$$

The field is essentially uniform as long as the distance from the center of the wall to the field point is much less than the dimensions of the wall.

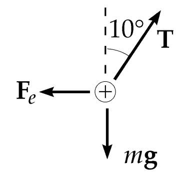
24.31 (a) $\boxed{E = 0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(32.0 \times 10^{-6})}{(0.200)^2} = \boxed{7.19 \text{ MN/C}}$$

24.32 The distance between centers is $2 \times 5.90 \times 10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{(46)^2 (1.60 \times 10^{-19} \text{ C})^2}{(2 \times 5.90 \times 10^{-15} \text{ m})^2} = 3.50 \times 10^3 \text{ N} = \boxed{3.50 \text{ kN}}$$

***24.33** Consider two balloons of diameter 0.2 m, each with mass 1 g, hanging apart with a 0.05 m separation on the ends of strings making angles of 10° with the vertical.



(a) $\Sigma F_y = T\cos 10° - mg = 0 \ \Rightarrow \ T = \dfrac{mg}{\cos 10°}$

$\Sigma F_x = T\sin 10° - F_e = 0 \ \Rightarrow \ F_e = T\sin 10°$, so

$$F_e = \left(\frac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10° = (0.001 \text{ kg})(9.8 \text{ m/s}^2)\tan 10°$$

$$F_e \approx 2 \times 10^{-3} \text{ N} \quad \boxed{\sim 10^{-3} \text{ N or } 1 \text{ mN}}$$

(b) $F_e = \dfrac{k_e q^2}{r^2}$

$$2 \times 10^{-3} \text{ N} \approx \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) q^2}{(0.25 \text{ m})^2}$$

$$q \approx 1.2 \times 10^{-7} \text{ C} \quad \boxed{\sim 10^{-7} \text{ C or } 100 \text{ nC}}$$

(c) $$E = \frac{k_e q}{r^2} \approx \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.2 \times 10^{-7} \text{ C})}{(0.25 \text{ m})^2} \approx 1.7 \times 10^4 \text{ N/C} \quad \boxed{\sim 10 \text{ kN/C}}$$

(d) $$\Phi_E = \frac{q}{\epsilon_0} \approx \frac{1.2 \times 10^{-7} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 1.4 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C} \quad \boxed{\sim 10 \text{ kN} \cdot \text{m}^2/\text{C}}$$

24.34 (a) $$\rho = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{5.70 \times 10^{-6}}{\frac{4}{3}\pi(0.0400)^3} = 2.13 \times 10^{-2} \text{ C / m}^3$$

$$q_{\text{in}} = \rho\left(\tfrac{4}{3}\pi r^3\right) = (2.13 \times 10^{-2})\left(\tfrac{4}{3}\pi\right)(0.0200)^3 = 7.13 \times 10^{-7} \text{ C} = \boxed{713 \text{ nC}}$$

(b) $$q_{\text{in}} = \rho\left(\tfrac{4}{3}\pi r^3\right) = (2.13 \times 10^{-2})\left(\tfrac{4}{3}\pi\right)(0.0400)^3 = \boxed{5.70 \ \mu\text{C}}$$

24.35 (a) $E = \dfrac{2k_e\lambda}{r} = \dfrac{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\left(2.00\times10^{-6}\ \text{C}\right)/7.00\ \text{m}\right]}{0.100\ \text{m}}$

$E = \boxed{51.4\ \text{kN/C, radially outward}}$

(b) $\Phi_E = EA\cos\theta = E(2\pi r\ell)\cos 0°$

$\Phi_E = \left(5.14\times10^{4}\ \text{N/C}\right)2\pi(0.100\ \text{m})(0.0200\ \text{m})(1.00) = \boxed{646\ \text{N}\cdot\text{m}^2/\text{C}}$

24.36 Note that the electric field in each case is directed radially inward, toward the filament.

(a) $E = \dfrac{2k_e\lambda}{r} = \dfrac{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C}\right)}{0.100\ \text{m}} = \boxed{16.2\ \text{MN/C}}$

(b) $E = \dfrac{2k_e\lambda}{r} = \dfrac{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C}\right)}{0.200\ \text{m}} = \boxed{8.09\ \text{MN/C}}$

(c) $E = \dfrac{2k_e\lambda}{r} = \dfrac{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C}\right)}{1.00\ \text{m}} = \boxed{1.62\ \text{MN/C}}$

24.37 $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{9.00\times10^{-6}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{508\ \text{kN/C, upward}}$

24.38 From Gauss's Law, $EA = \dfrac{Q}{\epsilon_0}$

$\sigma = \dfrac{Q}{A} = \epsilon_0 E = (8.85\times10^{-12})(130) = 1.15\times10^{-9}\ \text{C/m}^2 = \boxed{1.15\ \text{nC/m}^2}$

24.39 $\oint EdA = E(2\pi rl) = \dfrac{q_{in}}{\epsilon_0} \qquad E = \dfrac{q_{in}/l}{2\pi\epsilon_0 r} = \dfrac{\lambda}{2\pi\epsilon_0 r}$

(a) $r = 3.00$ cm $\boxed{E = 0}$ inside the conductor

(b) $r = 10.0$ cm $\quad E = \dfrac{30.0\times10^{-9}}{2\pi(8.85\times10^{-12})(0.100)} = \boxed{5400\ \text{N/C, outward}}$

(c) $r = 100$ cm $\quad E = \dfrac{30.0\times10^{-9}}{2\pi(8.85\times10^{-12})(1.00)} = \boxed{540\ \text{N/C, outward}}$

***24.40** Just above the aluminum plate (a conductor), the electric field is $E = \sigma'/\epsilon_0$ where the charge Q is divided equally between the upper and lower surfaces of the plate:

Thus $\sigma' = \dfrac{(Q/2)}{A} = \dfrac{Q}{2A}$ and $E = \dfrac{Q}{2\epsilon_0 A}$

For the glass plate (an insulator), $E = \sigma / 2\epsilon_0$ where $\sigma = Q / A$ since the entire charge Q is on the upper surface.

Therefore, $E = \dfrac{Q}{2\epsilon_0 A}$

The electric field at a point just above the center of the upper surface is the same for each of the plates.

$\boxed{E = \dfrac{Q}{2\epsilon_0 A}, \text{ vertically upward in each case (assuming } Q > 0)}$

***24.41** (a) $E = \sigma/\epsilon_0$ $\sigma = (8.00 \times 10^4)(8.85 \times 10^{-12}) = 7.08 \times 10^{-7}\ \text{C/m}^2$

$\boxed{\sigma = 708\ \text{nC/m}^2}$, positive on one face and negative on the other.

(b) $\sigma = \dfrac{Q}{A}$ $Q = \sigma A = (7.08 \times 10^{-7})\,(0.500)^2\ \text{C}$

$Q = 1.77 \times 10^{-7}\ \text{C} = \boxed{177\ \text{nC}}$, positive on one face and negative on the other.

24.42 Use Gauss's Law to evaluate the electric field in each region, recalling that the electric field is zero everywhere within conducting materials. The results are:

$\boxed{E = 0 \text{ inside the sphere and inside the shell}}$

$\boxed{E = k_e \dfrac{Q}{r^2} \text{ between sphere and shell, directed radially inward}}$

$\boxed{E = k_e \dfrac{2Q}{r^2} \text{ outside the shell, directed radially inward}}$

Charge $\boxed{-Q \text{ is on the outer surface of the sphere}}$.

Charge $\boxed{+Q \text{ is on the inner surface of the shell}}$, and $\boxed{+2Q \text{ is on the outer surface of the shell.}}$

24.43 The charge divides equally between the identical spheres, with charge $Q/2$ on each. Then they repel like point charges at their centers:

$$F = \frac{k_e(Q/2)(Q/2)}{(L+R+R)^2} = \frac{k_e\,Q^2}{4(L+2R)^2} = \frac{8.99\times10^9\ \text{N}\cdot\text{m}^2(60.0\times10^{-6}\ \text{C})^2}{4\ \text{C}^2(2.01\ \text{m})^2} = \boxed{2.00\ \text{N}}$$

***24.44** The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vise-versa. The local charge density and the electric field intensity are related by

$$E = \frac{\sigma}{\epsilon_0} \quad \text{or} \quad \sigma = \epsilon_0 E$$

(a) Where the radius of curvature is the greatest,

$$\sigma = \epsilon_0 E_{\text{min}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.80\times10^4\ \text{N/C}\right) = \boxed{248\ \text{nC/m}^2}$$

(b) Where the radius of curvature is the smallest,

$$\sigma = \epsilon_0 E_{\text{max}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(5.60\times10^4\ \text{N/C}\right) = \boxed{496\ \text{nC/m}^2}$$

24.45 (a) Inside surface: consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero, so the inside charge/length $= -\lambda$.

$$0 = \lambda\ell + q_{\text{in}} \Rightarrow \frac{q_{\text{in}}}{\ell} = \boxed{-\lambda}$$

Outside surface: The total charge on the metal cylinder is $2\lambda\ell = q_{in} + q_{out}$.

$$q_{out} = 2\lambda\ell + \lambda\ell$$

so the outside charge/length $= \boxed{3\lambda}$

(b) $$E = \frac{2k_e(3\lambda)}{r} = \frac{6k_e\,\lambda}{r} = \boxed{\frac{3\lambda}{2\pi\epsilon_0 r}}$$

24.46 (a) $E = \frac{k_e Q}{r^2} = \frac{\left(8.99 \times 10^9\right)\left(6.40 \times 10^{-6}\right)}{\left(0.150\right)^2} = \boxed{2.56\ \text{MN/C, radially inward}}$

(b) $\boxed{E = 0}$

24.47 (a) The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma = \frac{1}{2}\left(\frac{q}{A}\right) = \frac{1}{2}\frac{(4.00\times10^{-8}\text{ C})}{(0.500\text{ m})^2} = 8.00\times10^{-8}\text{ C / m}^2 = \boxed{80.0\text{ nC / m}^2}$$

(b) $$\mathbf{E} = \left(\frac{\sigma}{\epsilon_0}\right)\mathbf{k} = \left(\frac{8.00\times10^{-8}\text{ C / m}^2}{8.85\times10^{-12}\text{ C}^2\text{ / N}\cdot\text{m}^2}\right)\mathbf{k} = \boxed{(9.04\text{ kN / C})\mathbf{k}}$$

(c) $$\mathbf{E} = \boxed{(-9.04\text{ kN / C})\mathbf{k}}$$

24. 48 (a) The charge $+q$ at the center induces charge $-q$ on the inner surface of the conductor, where its surface density is:

$$\sigma_a = \boxed{\frac{-q}{4\pi a^2}}$$

(b) The outer surface carries charge $Q+q$ with density

$$\sigma_b = \boxed{\frac{Q+q}{4\pi b^2}}$$

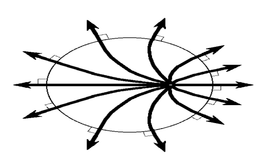
24.49 (a) $\boxed{E=0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{(8.99\times10^9)(8.00\times10^{-6})}{(0.0300)^2} = 7.99\times10^7\text{ N / C} = \boxed{79.9\text{ MN/C}}$$

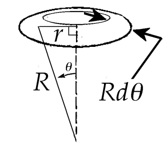
(c) $\boxed{E=0}$

(d) $$E = \frac{k_e Q}{r^2} = \frac{(8.99\times10^9)(4.00\times10^{-6})}{(0.0700)^2} = 7.34\times10^6\text{ N / C} = \boxed{7.34\text{ MN/C}}$$

24.50 An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.



24.51 (a) Uniform **E**, pointing radially outward, so $\Phi_E = EA$. The arc length is $ds = R\,d\theta$, and the circumference is $2\pi r = 2\pi R \sin\theta$



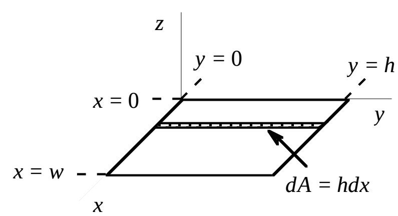
$$A = \int 2\pi r\,ds = \int_0^{\theta} (2\pi R \sin\theta) R\,d\theta = 2\pi R^2 \int_0^{\theta} \sin\theta\,d\theta = 2\pi R^2 (-\cos\theta)\Big|_0^{\theta} = 2\pi R^2 (1-\cos\theta)$$

$$\Phi_E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\cdot 2\pi R^2(1-\cos\theta) = \boxed{\frac{Q}{2\epsilon_0}(1-\cos\theta)}$$ [independent of R!]

(b) For $\theta = 90.0°$ (hemisphere): $\Phi_E = \frac{Q}{2\epsilon_0}(1-\cos 90°) = \boxed{\frac{Q}{2\epsilon_0}}$

(c) For $\theta = 180°$ (entire sphere): $\Phi_E = \frac{Q}{2\epsilon_0}(1-\cos 180°) = \boxed{\frac{Q}{\epsilon_0}}$ [Gauss's Law]

***24.52** In general, $\mathbf{E} = ay\,\mathbf{i} + bz\,\mathbf{j} + cx\,\mathbf{k}$



In the xy plane, $z = 0$ and $\mathbf{E} = ay\,\mathbf{i} + cx\,\mathbf{k}$

$$\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = \int (ay\,\mathbf{i} + cx\,\mathbf{k})\cdot \mathbf{k}\,dA$$

$$\Phi_E = ch\int_{x=0}^{w} x\,dx = ch\frac{x^2}{2}\Big|_{x=0}^{w} = \boxed{\frac{chw^2}{2}}$$

***24.53** (a) $q_{in} = +3Q - Q = \boxed{+2Q}$

(b) The charge distribution is spherically symmetric and $q_{in} > 0$. Thus, the field is directed $\boxed{\text{radially outward}}$.

(c) $E = \frac{k_e q_{in}}{r^2} = \boxed{\frac{2k_e Q}{r^2}}$ for $r \ge c$

(d) Since all points within this region are located inside conducting material, $\boxed{E = 0}$ for $b < r < c$.

(e) $\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = 0 \;\Rightarrow\; q_{in} = \epsilon_0 \Phi_E = \boxed{0}$

(f) $q_{in} = \boxed{+3Q}$

(g) $E = \frac{k_e q_{in}}{r^2} = \boxed{\frac{3k_e Q}{r^2}}$ (radially outward) for $a \le r < b$

(h) $q_{\text{in}} = \rho V = \left(\frac{+3Q}{\frac{4}{3}\pi a^3}\right)\left(\frac{4}{3}\pi r^3\right) = \boxed{+3Q\frac{r^3}{a^3}}$

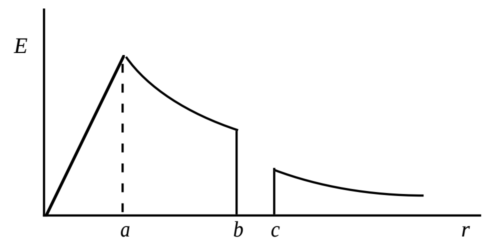
(i) $E = \frac{k_e q_{\text{in}}}{r^2} = \frac{k_e}{r^2}\left(+3Q\frac{r^3}{a^3}\right) = \boxed{3k_e Q\frac{r}{a^3}}$ (radially outward) for $0 \le r \le a$

(j) From part (d), $E = 0$ for $b < r < c$. Thus, for a spherical gaussian surface with $b < r < c$, $q_{\text{in}} = +3Q + q_{\text{inner}} = 0$ where q_{inner} is the charge on the inner surface of the conducting shell. This yields $q_{\text{inner}} = \boxed{-3Q}$

(k) Since the total charge on the conducting shell is $q_{\text{net}} = q_{\text{outer}} + q_{\text{inner}} = -Q$, we have

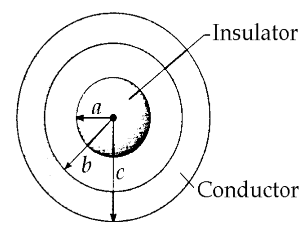
$q_{\text{outer}} = -Q - q_{\text{inner}} = -Q - (-3Q) = \boxed{+2Q}$

(l) This is shown in the figure to the right.



24.54 The sphere with large charge creates a strong field to polarize the other sphere. That means it pushes the excess charge over to the far side, leaving charge of the opposite sign on the near side. This patch of opposite charge is smaller in amount but located in a stronger external field, so it can feel a force of attraction that is larger than the repelling force felt by the larger charge in the weaker field on the other side.

24.55 (a) $\oint \mathbf{E} \cdot d\mathbf{A} = E(4\pi r^2) = q_{in}/\text{e}_0$



For $r < a$, $q_{in} = \rho\left(\frac{4}{3}\pi r^3\right)$ so $\boxed{E = \frac{pr}{3\text{e}_0}}$

For $a < r < b$ and $c < r$, $q_{\text{in}} = Q$ so that $\boxed{E = \frac{Q}{4\pi r^2 \text{e}_0}}$

For $b \le r \le c$, $E = 0$, since $\boxed{E = 0}$ inside a conductor.

(b) Let q_1 = induced charge on the inner surface of the hollow sphere. Since $E = 0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b \le r \le c$ must be zero.

Therefore, $q_1 + Q = 0$ and $\sigma_1 = \frac{q_1}{4\pi b^2} = \boxed{\frac{-Q}{4\pi b^2}}$

Let q_2 = induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require $q_1 + q_2 = 0$

and $\sigma_2 = \frac{q_1}{4\pi c^2} = \boxed{\frac{Q}{4\pi c^2}}$

24.56 $\oint \mathbf{E}\cdot d\mathbf{A} = E\left(4\pi r^2\right) = \dfrac{q_{\text{in}}}{\epsilon_0}$

(a) $\left(-3.60\times10^{3}\ \text{N/C}\right)4\pi(0.100\ \text{m})^2 = \dfrac{Q}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}$ $\qquad (a < r < b)$

$Q = -4.00\times10^{-9}\ \text{C} = \boxed{-4.00\ \text{nC}}$

(b) We take Q' to be the net charge on the hollow sphere. Outside c,

$\left(+2.00\times10^{2}\ \text{N/C}\right)4\pi(0.500\ \text{m})^2 = \dfrac{Q+Q'}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}$ $\qquad (r > c)$

$Q + Q' = +5.56\times10^{-9}\ \text{C}$, so $Q' = +9.56\times10^{-9}\ \text{C} = \boxed{+9.56\ \text{nC}}$

(c) For $b < r < c$: $E = 0$ and $q_{\text{in}} = Q + Q_1 = 0$ where Q_1 is the total charge on the inner surface of the hollow sphere. Thus, $Q_1 = -Q = \boxed{+4.00\ \text{nC}}$

Then, if Q_2 is the total charge on the outer surface of the hollow sphere, $Q_2 = Q' - Q_1 = 9.56\ \text{nC} - 4.00\ \text{nC} = \boxed{+5.56\ \text{nC}}$

24.57 The field direction is radially outward perpendicular to the axis. The field strength depends on r but not on the other cylindrical coordinates θ or z. Choose a Gaussian cylinder of radius r and length L. If $r < a$,

$\Phi_E = \dfrac{q_{in}}{\epsilon_0}$ and $E\left(2\pi rL\right) = \dfrac{\lambda L}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi r\epsilon_0}$ or $\boxed{\mathbf{E} = \dfrac{\lambda}{2\pi r\epsilon_0}\quad (r < a)}$

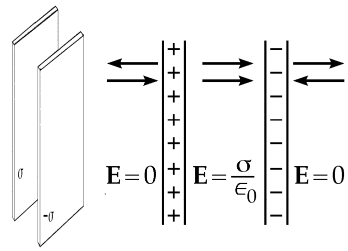
If $a < r < b$, $E\left(2\pi rL\right) = \dfrac{\lambda L + \rho\pi\left(r^2 - a^2\right)L}{\epsilon_0}$

$\boxed{\mathbf{E} = \dfrac{\lambda + \rho\pi\left(r^2 - a^2\right)}{2\pi r\epsilon_0}\quad (a < r < b)}$

If $r > b$, $E\left(2\pi rL\right) = \dfrac{\lambda L + \rho\pi\left(b^2 - a^2\right)L}{\epsilon_0}$

$$\mathbf{E} = \frac{\lambda + \rho\pi\left(b^2 - a^2\right)}{2\pi r \epsilon_0} \qquad (r > b)$$

24.58 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by Equation 24.8:



$$|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0}$$

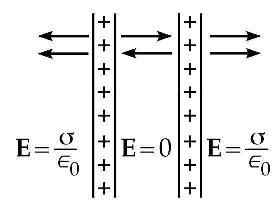
(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\boxed{E = 0}$.

(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\boxed{E = \frac{\sigma}{\epsilon_0} \text{ toward the right}}$$

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\boxed{E = 0}$.

24.59 The magnitude of the field due to each sheet given by Equation 24.8 is



$$E = \frac{\sigma}{2\epsilon_0} \text{ directed perpendicular to the sheet.}$$

(a) In the region to the left of the pair of sheets, both fields are directed toward the left and the net field is

$$E = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the left}}$$

(b) In the region between the sheets, the fields due to the individual sheets are oppositely directed and the net field is

$$E = \boxed{0}$$

(c) In the region to the right of the pair of sheets, both fields are directed toward the right and the net field is

$$E = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

Goal Solution

Repeat the calculations for Problem 58 when both sheets have **positive** uniform charge densities of value σ. **Note:** The new problem statement would be as follows: Two infinite, nonconducting sheets of charge are parallel to each other, as shown in Figure P24.58. Both sheets have positive uniform charge densities σ. Calculate the value of the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets.

G: When both sheets have the same charge density, a positive test charge at a point midway between them will experience the same force in opposite directions from each sheet. Therefore, the electric field here will be zero. (We should ask: can we also conclude that the electron will experience equal and oppositely directed forces *everywhere* in the region between the plates?)

Outside the sheets the electric field will point away and should be twice the strength due to one sheet of charge, so $E = \sigma / \epsilon_0$ in these regions.

O: The principle of superposition can be applied to add the electric field vectors due to each sheet of charge.

A: For each sheet, the electric field at any point is $|\mathbf{E}| = \sigma/(2\epsilon_0)$ directed away from the sheet.

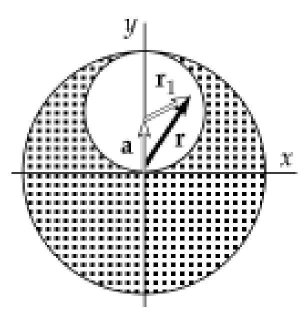
(a) At a point to the left of the two parallel sheets $\quad \mathbf{E} = E_1(-\mathbf{i}) + E_2(-\mathbf{i}) = 2E(-\mathbf{i}) = -\dfrac{\sigma}{\epsilon_0}\mathbf{i}$

(b) At a point between the two sheets $\quad \mathbf{E} = E_1\mathbf{i} + E_2(-\mathbf{i}) = 0$

(c) At a point to the right of the two parallel sheets $\quad \mathbf{E} = E_1\mathbf{i} + E_2\mathbf{i} = 2E\mathbf{i} = \dfrac{\sigma}{\epsilon_0}\mathbf{i}$

L: We essentially solved this problem in the Gather information step, so it is no surprise that these results are what we expected. A better check is to confirm that the results are complementary to the case where the plates are oppositely charged (Problem 58).

24.60 The resultant field within the cavity is the superposition of two fields, one $\mathbf{E}_+$ due to a uniform sphere of positive charge of radius $2a$, and the other $\mathbf{E}_-$ due to a sphere of negative charge of radius a centered within the cavity.



$$\frac{4}{3}\frac{\pi r^3 \rho}{\epsilon_0} = 4\pi r^2 E_+ \quad \text{so} \quad \mathbf{E}_+ = \frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}} = \frac{\rho \mathbf{r}}{3\epsilon_0}$$

$$-\frac{4}{3}\frac{\pi r_1^3 \rho}{\epsilon_0} = 4\pi r_1^2 E_- \quad \text{so} \quad \mathbf{E}_- = \frac{\rho r_1}{3\epsilon_0}(-\hat{\mathbf{r}}_1) = \frac{-\rho}{3\epsilon_0}\mathbf{r}_1$$

Since $\mathbf{r} = \mathbf{a} + \mathbf{r}_1$, $\quad \mathbf{E}_- = \dfrac{-\rho(\mathbf{r} - \mathbf{a})}{3\epsilon_0}$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{\rho \mathbf{r}}{3\epsilon_0} - \frac{\rho \mathbf{r}}{3\epsilon_0} + \frac{\rho \mathbf{a}}{3\epsilon_0} = \frac{\rho \mathbf{a}}{3\epsilon_0} = 0\mathbf{i} + \frac{\rho a}{3\epsilon_0}\mathbf{j}$$

Thus, $\boxed{E_x = 0}$ and $\boxed{E_y = \dfrac{\rho a}{3\epsilon_0}}$ at all points within the cavity.

24.61 First, consider the field at distance $r < R$ from the center of a uniform sphere of positive charge $(Q = +e)$ with radius R.

$$\left(4\pi r^2\right)E = \frac{q_{in}}{\epsilon_0} = \frac{\rho V}{\epsilon_0} = \left(\frac{+e}{\frac{4}{3}\pi R^3}\right)\frac{\frac{4}{3}\pi r^3}{\epsilon_0} \quad \text{so} \quad E = \left(\frac{e}{4\pi\epsilon_0 R^3}\right)r \text{ directed outward}$$

(a) The force exerted on a point charge $q = -e$ located at distance r from the center is then

$$F = qE = -e\left(\frac{e}{4\pi\epsilon_0 R^3}\right)r = -\left(\frac{e^2}{4\pi\epsilon_0 R^3}\right)r = \boxed{-Kr}$$

(b) $K = \dfrac{e^2}{4\pi\epsilon_0 R^3} = \boxed{\dfrac{k_e e^2}{R^3}}$

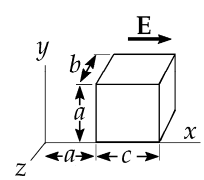
(c) $F_r = m_e a_r = -\left(\dfrac{k_e e^2}{R^3}\right)r$, so $a_r = -\left(\dfrac{k_e e^2}{m_e R^3}\right)r = -\omega^2 r$

Thus, the motion is simple harmonic with frequency $f = \dfrac{\omega}{2\pi} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e R^3}}}$

(d) $f = 2.47 \times 10^{15}\text{ Hz} = \dfrac{1}{2\pi}\sqrt{\dfrac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)R^3}}$

which yields $R^3 = 1.05\times10^{-30}\text{ m}^3$, or $R = 1.02\times10^{-10}\text{ m} = \boxed{102\text{ pm}}$

24.62 The electric field throughout the region is directed along x; therefore, **E** will be perpendicular to dA over the four faces of the surface which are perpendicular to the yz plane, and E will be parallel to dA over the two faces which are parallel to the yz plane. Therefore,



$$\Phi_E = -\left(E_x\big|_{x=a}\right)A + \left(E_x\big|_{x=a+c}\right)A = -\left(3 + 2a^2\right)ab + \left(3 + 2(a+c)^2\right)ab = 2abc(2a+c)$$

Substituting the given values for a, b, and c, we find $\Phi_E = \boxed{0.269\text{ N}\cdot\text{m}^2/\text{C}}$

$Q = \epsilon_0\Phi_E = 2.38\times10^{-12}\text{ C} = \boxed{2.38\text{ pC}}$

24.63 $\oint \mathbf{E}\cdot d\mathbf{A} = E(4\pi r^2) = \dfrac{q_{in}}{\epsilon_0}$

(a) For $r > R$, $q_{in} = \displaystyle\int_0^R Ar^2(4\pi r^2)dr = 4\pi\frac{AR^5}{5}$ and $E = \boxed{\dfrac{AR^5}{5\epsilon_0 r^2}}$

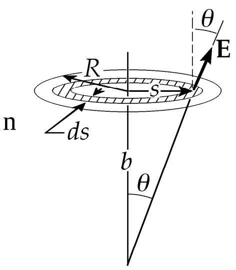
(b) For $r < R$, $q_{in} = \displaystyle\int_0^r Ar^2(4\pi r^2)dr = \frac{4\pi Ar^5}{5}$ and $E = \boxed{\dfrac{Ar^3}{5\epsilon_0}}$

24.64 The total flux through a surface enclosing the charge Q is Q/ϵ_0. The flux through the disk is

$$\Phi_{\text{disk}} = \int \mathbf{E} \cdot d\mathbf{A}$$

where the integration covers the area of the disk. We must evaluate this integral and set it equal to $\frac{1}{4}\,Q/\epsilon_0$ to find how b and R are related. In the figure, take $d\mathbf{A}$ to be the area of an annular ring of radius s and width ds. The flux through $d\mathbf{A}$ is

$$\mathbf{E} \cdot d\mathbf{A} = E\,dA\cos\theta = E(2\pi\,s ds)\cos\theta$$



The magnitude of the electric field has the same value at all points within the annular ring,

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{s^2+b^2} \qquad \text{and} \qquad \cos\theta = \frac{b}{r} = \frac{b}{(s^2+b^2)^{1/2}}$$

Integrate from $s = 0$ to $s = R$ to get the flux through the entire disk.

$$\Phi_{E,\text{ disk}} = \frac{Qb}{2\epsilon_0}\int_0^R \frac{s\,ds}{(s^2+b^2)^{3/2}} = \frac{Qb}{2\epsilon_0}\left[-(s^2+b^2)^{1/2}\right]\Bigg|_0^R = \frac{Q}{2\epsilon_0}\left[1 - \frac{b}{(R^2+b^2)^{1/2}}\right]$$

The flux through the disk equals $Q/4\epsilon_0$ provided that $\dfrac{b}{(R^2+b^2)^{1/2}} = \dfrac{1}{2}$.

This is satisfied if $\boxed{R = \sqrt{3}\,b}$.

24.65

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{1}{\epsilon_0}\int_0^r \frac{a}{r}4\pi r^2 dr$$

$$E4\pi r^2 = \frac{4\pi a}{\epsilon_0}\int_0^r r\,dr = \frac{4\pi a}{\epsilon_0}\frac{r^2}{2}$$

$$\boxed{E = \frac{a}{2\epsilon_0}} \quad = \text{constant magnitude}$$

(The direction is radially outward from center for positive a; radially inward for negative a.)

24.66 In this case the charge density is *not uniform*, and Gauss's law is written as $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0}\int \rho \, dV$.

We use a gaussian surface which is a cylinder of radius r, length ℓ, and is coaxial with the charge distribution.

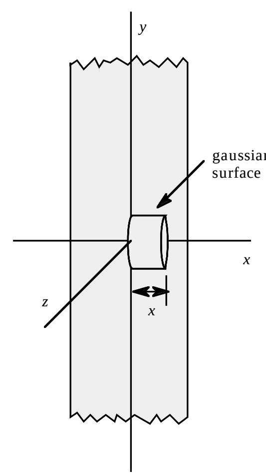
(a) When $r < R$, this becomes $E(2\pi r \ell) = \frac{\rho_0}{\epsilon_0}\int_0^r \left(a - \frac{r}{b}\right) dV$. The element of volume is a cylindrical shell of radius r, length ℓ, and thickness dr so that $dV = 2\pi r \ell \, dr$.

$$E(2\pi r \ell) = \left(\frac{2\pi r^2 \ell \rho_0}{\epsilon_0}\right)\left(\frac{a}{2} - \frac{r}{3b}\right)$$ so inside the cylinder, $$\boxed{E = \frac{\rho_0 r}{2\epsilon_0}\left(a - \frac{2r}{3b}\right)}$$

(b) When $r > R$, Gauss's law becomes

$$E(2\pi r \ell) = \frac{\rho_0}{\epsilon_0}\int_0^R \left(a - \frac{r}{b}\right)(2\pi r \ell \, dr)$$ or outside the cylinder, $$\boxed{E = \frac{\rho_0 R^2}{2\epsilon_0 r}\left(a - \frac{2R}{3b}\right)}$$

24.67 (a) Consider a cylindrical shaped gaussian surface perpendicular to the yz plane with one end in the yz plane and the other end containing the point x:



Use Gauss's law: $$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

By symmetry, the electric field is zero in the yz plane and is perpendicular to $d\mathbf{A}$ over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap containing the point x :

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} \quad \text{or} \quad EA = \frac{\rho(Ax)}{\epsilon_0}$$

so that at distance x from the mid-line of the slab, $$\boxed{E = \frac{\rho x}{\epsilon_0}}$$

(b) $$a = \frac{F}{m_e} = \frac{(-e)E}{m_e} = -\left(\frac{\rho e}{m_e \epsilon_0}\right)x$$

The acceleration of the electron is of the form $a = -\omega^2 x$ with $\omega = \sqrt{\frac{\rho e}{m_e \epsilon_0}}$

Thus, the motion is simple harmonic with frequency $$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{\rho e}{m_e \epsilon_0}}}$$

24.68 Consider the gaussian surface described in the solution to problem 67.

(a) For $x > \frac{d}{2}$, $dq = \rho\, dV = \rho A\, dx = C A x^2\, dx$

$$\int \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} \int dq$$

$$EA = \frac{CA}{\epsilon_0} \int_0^{d/2} x^2\, dx = \frac{1}{3}\left(\frac{CA}{\epsilon_0}\right)\left(\frac{d^3}{8}\right)$$

$$E = \frac{Cd^3}{24\epsilon_0} \quad \text{or} \quad \boxed{\mathbf{E} = \frac{Cd^3}{24\epsilon_0}\mathbf{i} \text{ for } x > \frac{d}{2}; \qquad \mathbf{E} = -\frac{Cd^3}{24\epsilon_0}\mathbf{i} \text{ for } x < -\frac{d}{2}}$$

(b) For $-\frac{d}{2} < x < \frac{d}{2}$ $\quad \int \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} \int dq = \frac{CA}{\epsilon_0} \int_0^x x^2\, dx = \frac{CAx^3}{3\epsilon_0}$

$$\boxed{\mathbf{E} = \frac{Cx^3}{3\epsilon_0}\mathbf{i} \text{ for } x > 0; \qquad \mathbf{E} = -\frac{Cx^3}{3\epsilon_0}\mathbf{i} \text{ for } x < 0}$$

24.69 (a) A point mass m creates a gravitational acceleration $\mathbf{g} = -\frac{Gm}{r^2}\hat{\mathbf{r}}$ at a distance r.

The flux of this field through a sphere is $\oint \mathbf{g} \cdot d\mathbf{A} = -\frac{Gm}{r^2}\left(4\pi r^2\right) = -4\pi Gm$

Since the r has divided out, we can visualize the field as unbroken field lines. The same flux would go through any other closed surface around the mass. If there are several or no masses inside a closed surface, each creates field to make its own contribution to the net flux according to

$$\boxed{\oint \mathbf{g} \cdot d\mathbf{A} = -4\pi G m_{\text{in}}}$$

(b) Take a spherical gaussian surface of radius r. The field is inward so

$$\oint \mathbf{g} \cdot d\mathbf{A} = g\, 4\pi r^2 \cos 180° = -g\, 4\pi r^2$$

and $\quad -4\pi G m_{\text{in}} = -4G\frac{4}{3}\pi r^3 \rho$

Then, $-g\, 4\pi r^2 = -4\pi G \frac{4}{3}\pi r^3 \rho$ and $g = \frac{4}{3}\pi r \rho G$

Or, since $\rho = M_E / \frac{4}{3}\pi R_E^3$, $\quad g = \frac{M_E G r}{R_E^3}$ or $\boxed{\mathbf{g} = \frac{M_E G r}{R_E^3} \text{ inward}}$